



DEPARTMENT OF EDUCATION

GRADE 10

MATHEMATICS

UNIT 5

Trigonometry

Label the sides of the triangle

opposite, adjacent, hypotenuse

Pythagoras' Theorem

$$A^2 + B^2 = C^2$$

$x^2 = 9^2 + 7^2$
 $x^2 = 81 + 49$
 $x^2 = 130$
 $x = \sqrt{130} = 11.4$

Trigonometry

sin = opp/hyp

cos = adj/hyp

Tan x = opp/adj

Finding a side

$\tan A = \frac{\text{opp}}{\text{adj}}$
 $\tan 35^\circ = \frac{x}{8}$
 $8 \times \tan 35^\circ = x$
 $5.6016603 \times 8 = x$
 $x = 44.8132824$

Finding an angle

$\tan x = \frac{\text{opp}}{\text{adj}}$
 $\tan x = \frac{3}{8} = 0.375$
 $x = \tan^{-1} 0.375$
 $x = 20.556045^\circ$

The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Area of a triangle = $\frac{1}{2} ab \sin C$

Remember to use the formulae properly when you solve problems!

TRIGONOMETRY

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2016

GRADE 10

MATHEMATICS

UNIT 5

TRIGONOMETRY

TOPIC 1: ANGLES AND TRIANGLES

TOPIC 2: PYTHAGORAS' THEOREM

TOPIC 3: POLYGONS AND THE CIRCLES

**TOPIC 4: INTRODUCTION TO TRIGONOMETRIC
RATIOS**

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Academic Advisory Committee-FODE

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MR. DEMAS TONGOGO
Principal-FODE



Flexible Open and Distance Education
Papua New Guinea

Published in 2016

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National OBE Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

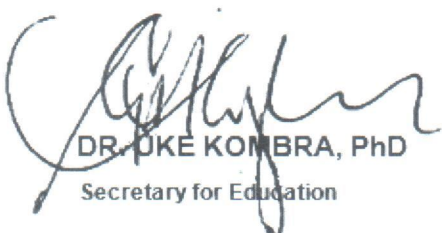
Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.



DR. DUKE KOMBRA, PhD
Secretary for Education

UNIT 5: TRIGONOMETRY



Dear Student

This is the fifth unit of the Grade 10 Mathematics Course. This unit is based on the National Department of Education Lower Secondary Syllabus and Curriculum Framework for Grade 10.

This Unit consists of four topics:

- Topic 1: Angles and Triangles**
- Topic 2: Pythagoras' Theorem**
- Topic 3: Polygons and the Circle**
- Topic 4: Introduction to Trigonometric Ratios**

In Topic 1- Angles and Triangles – You will revise angle properties of intersecting lines, parallel lines and triangles. You will also learn to solve real – life related problems involving angle relationships and similar triangles.

In Topic 2- Pythagoras' Theorem – You will revise squares and square roots and learn to use its table to find the squares and square root of a given number. The history of the Pythagoras' Theorem will also be discussed in this topic. Since this topic is all about the theorem, practical and real – life related problems involving the theorem are also presented in this topic.

In Topic 3- Polygons and the Circle – You will learn about quadrilaterals particularly parallelograms. You will calculate Interior angle sum of a polygon and use deductive reasoning to solve problems. Circle and Tangent Theorems and their properties will also be covered in this topic.

In Topic 4- Introduction to Trigonometric Ratios – You will define what Trigonometry is based on its etymology (origin), learn how to formulate the sine, cosine and tangent ratios. This topic will give you the fundamentals of Trigonometry which are very important in the development of higher Trigonometry and related courses in Unit 6 and Grades 11 and 12.

You will also learn in this unit ways on how to come up with formulas, strategies to make learning Mathematics fun and easy.

You will find that each lesson has reading material to study, worked examples and a practice exercise. The answers to the practice exercises are given at the end of each topic.

All the lessons are written in simple language with comic characters to guide you. The practice exercises are graded to help you learn the process of working out problems.

We hope you enjoy learning this unit.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Unit.

- Step 1: Start with TOPIC 1 Lesson 1 and work through it.
- Step 2: When you complete Lesson 1, do Practice Exercise 1.
- Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of the TOPIC 1.
- Step 4: Then, revise Lesson 1 and correct your mistakes, if any.
- Step 5: When you have completed all these steps, tick the check-box for Lesson, on the Contents Page (page 3) like this:

☒ Lesson 1: Directed numbers in Practical Situations

Then go on to the next Lesson. Repeat the same process until you complete all of the lessons in Topic 1.

As you complete each lesson, tick the check-box for that lesson, on the Content Page 3, like this ☒. This helps you to check on your progress.

- Step 6: Revise the Topic using Topic 1 Summary, then, do Topic test 1 in Assignment 5.

Then go on to the next Topic. Repeat the same process until you complete all of the four Topics in Unit 5.

Assignment: (Four Topics and a Unit Test)

When you have revised each Topic using the Topic Summary, do the Topic Test in your Assignment. The Unit book tells you when to do each Topic Test.

When you have completed the four Topic Tests, revise well and do the Unit Test. The Assignment tells you when to do the Unit Test.

The Topic Tests and the Unit test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment will count towards your final mark. If you score less than 50%, you will repeat that Assignment.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and make sure that you pass all of the Assignments.

TOPIC 1

ANGLES AND TRIANGLES

- | | |
|------------------|--|
| Lesson 1: | Angles and Angle Properties of Intersecting Lines |
| Lesson 2: | Angle Properties of Parallel Lines |
| Lesson 3: | Angle Properties of Triangles |
| Lesson 4: | Similar Triangles |

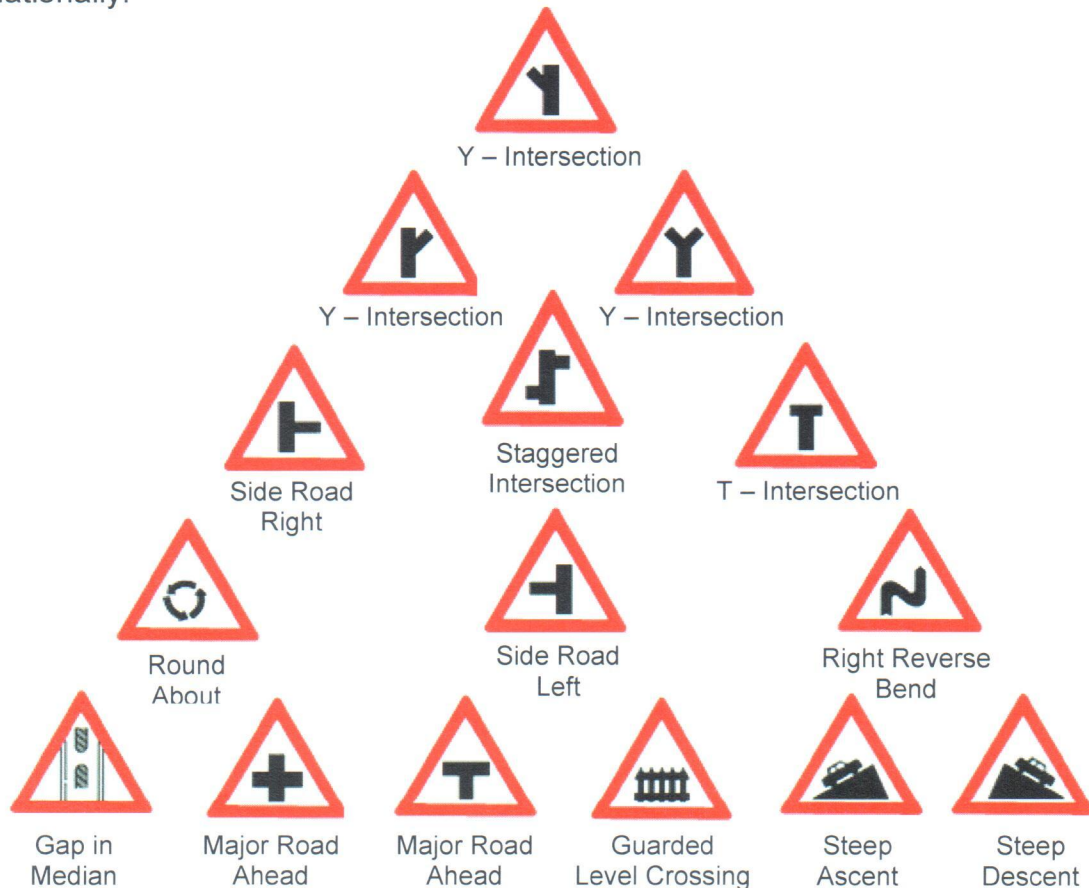
TOPIC 1: ANGLES AND TRIANGLES

Introduction



Have you come across the road signs below?

Road safety must always be considered when driving. To make sure that you are safe while on the road, you should be familiar with the different road signs. Below are some examples of the common road signs used internationally.



Some of these signs may be familiar to you because they are widely used in the country. Such signs include the roundabout and the right reverse bend that can be found around the country.

In this Topic, you will learn about intersecting lines and their properties. These lines make most of the road signs shown above. A line as one of the basis of geometry plays an important role in applying other geometrical concepts. This knowledge will be the basis in understanding such road signs above.

The topic also includes discussion about triangles, its classifications and properties. The triangle is considered as the simplest among all polygons. Each polygon can be divided to form several triangles. Thus, thorough understanding of its characteristics allows us to apply the ideas in real world problems.

Lesson 1: Angles and Angle Properties of Intersecting Lines



You learnt something about Angles in Strand 2 of Grade 8 Mathematics.



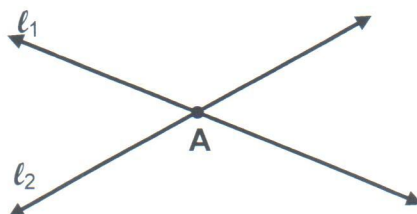
In this lesson you will:

- differentiate intersecting lines from non-intersecting lines
- identify the angle properties of intersecting lines
- solve problems involving intersecting lines.

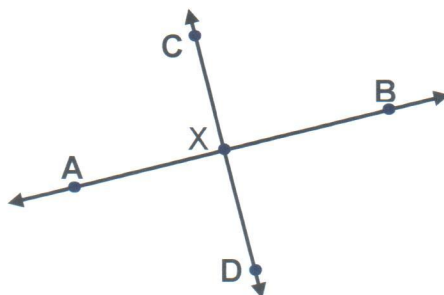
Intersecting Lines

Two lines that meet at a common point called **point of intersection** are known as **Intersecting Lines**.

In the figure below, ℓ_1 (read as line 1) and ℓ_2 (read as line 2) are intersecting lines. Their point of intersection is at point A.



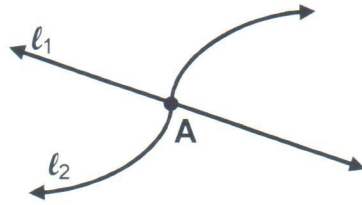
The next figure, shows that $\overleftrightarrow{AB}$ (read as line AB) intersects $\overleftrightarrow{CD}$ (read as line CD) at X (read as point X)



Notice that in the figure below, $\ell_1 \parallel \ell_2$ (read as line 1 is parallel with line 2), therefore there is **no point of intersection**. These lines are not intersecting lines.



The next pair is also not considered as intersecting lines. Why?



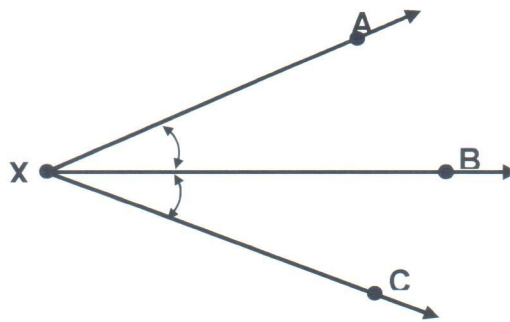
Obviously, l_1 intersects a curve (denoted as l_2).

Angle Relationships

Adjacent Angles

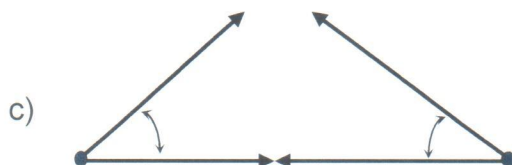
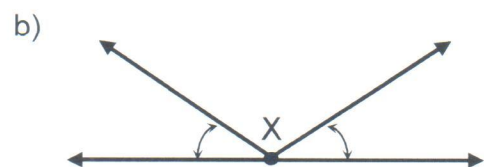
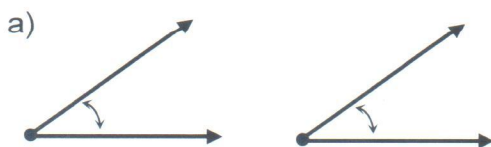
Two angles are said to be **adjacent** if they share a common side and a common vertex.

In the figure, $\angle AXB$ is adjacent with $\angle BXC$. Their common side is $\overrightarrow{XB}$ (read as ray XB) and their common vertex is at X (read as point X).



Example 1

Are the following pairs of angles adjacent?



Solution:

- a) No. There is no common side and no common endpoint.
- b) No. Although they have the same vertex at X, they do not share a common side.
- c) No. There is no common side and no common endpoint.
- d) Yes. They share a common vertex X and a common side $\overrightarrow{XY}$.

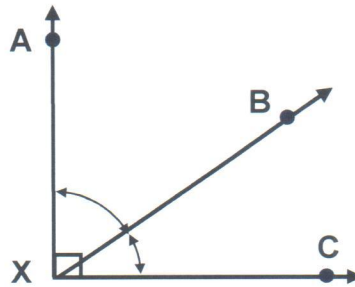
Complementary Angles

Two angles are said to be **complementary** if the sum of their measures is **exactly 90°** .

Below, $\angle ABC$ measures 50° while $\angle XYZ$ measures 40° . If we will get the sum of their measures we have $50^\circ + 40^\circ = 90^\circ$. Since the sum of their measure is exactly 90° , we consider the two angles as complementary angles.



In the next figure, we consider the two angles as complementary although there are no measures indicated.



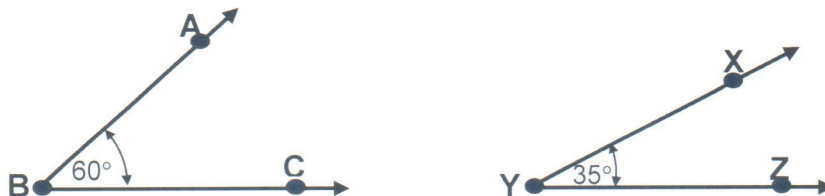
How come?

Observe that the two angles are adjacent and they form a right angle. You have learnt in your previous Lessons that a right angle measures exactly 90° . Hence, we consider $\angle AXB$ and $\angle BXC$ as complementary because they form the right angle AXC .

Example 2

Are the following pairs of angles complementary?

a)



b) $m\angle CHE = 25^\circ$, $m\angle JOS = 45^\circ$.

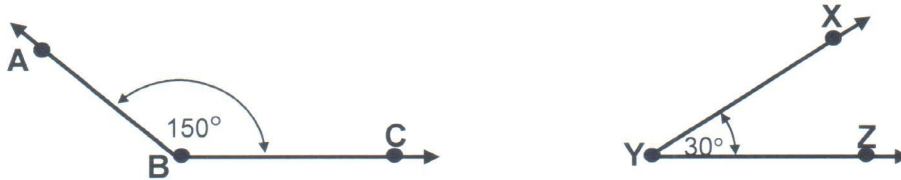
Solution

- a) No. Because $60^\circ + 35^\circ = 95^\circ$. Their sum is greater than 90° .
 b) No. Because $25^\circ + 45^\circ = 70^\circ$. Their sum is less than 90° .

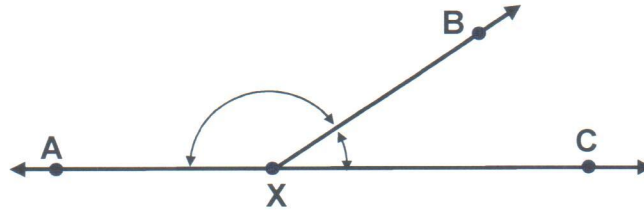
Supplementary Angles

Two angles are said to be **supplementary** if the sum of their measures is **exactly 180°** .

Below, $\angle ABC$ measures 150° while $\angle XYZ$ measures 30° . If we will get the sum of their measures we have $150^\circ + 30^\circ = 180^\circ$. Since the sum of their measures is exactly 180° , we consider the two angles as supplementary angles.



In the next figure, we consider the two angles as supplementary although there are no measures indicated.



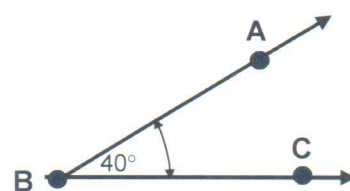
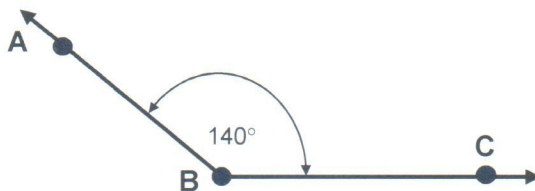
How come?

Observe that the two angles are adjacent and they form a straight angle. You have learnt in your previous Lessons that a straight angle measures exactly 180° . Hence, we consider $\angle AXB$ and $\angle BXC$ as supplementary because they form the straight angle $\overline{AXC}$ or could also be called as $\overline{AC}$.

Example 3

Are the following pairs of angles supplementary?

a)



b) a right angle and an obtuse angle

c) two right angles

Solution:

a) Yes. Because $140^\circ + 40^\circ = 180^\circ$. Their sum is exactly 180° .

b) No. Because a right angle measures exactly 90° while an obtuse angle measures greater than 90° . The sum of the measure of their respective angles would be greater than 180° .

- c) Yes. Since the measure of a right angle is exactly 90° . Getting the sum of the two yields to $90^\circ + 90^\circ = 180^\circ$.

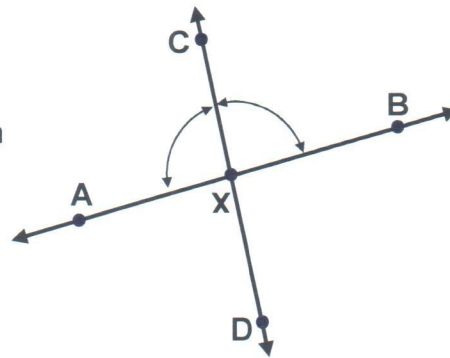
Did you notice that in a pair of supplementary angles, one is an obtuse (measuring more than 90°) while the other is an acute (measuring less than 90°). Two acute angles cannot be supplementary because their sum will definitely be less than 180° . It is also not possible to have two obtuse angles in a supplementary pair because it will definitely give a sum more than 180° .

Angle Relationships Formed by Intersecting Lines

It is very interesting to note that two intersecting lines form 4 angles. And out of these angles, special angle relationships are formed.

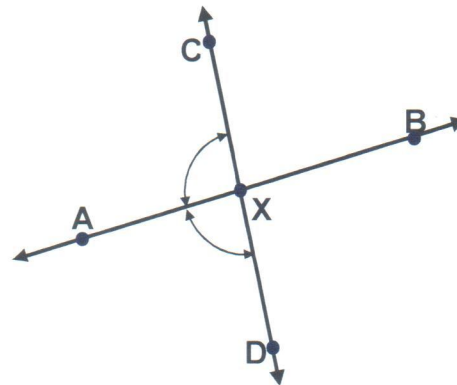
In the figure, $\angle CXA$ is adjacent with $\angle CXB$.

Their common side is $\overrightarrow{XC}$ and their common vertex is X .



Likewise $\angle CXA$ is also adjacent with $\angle AXD$.

Their common side is $\overrightarrow{XA}$ and their common Vertex is X .



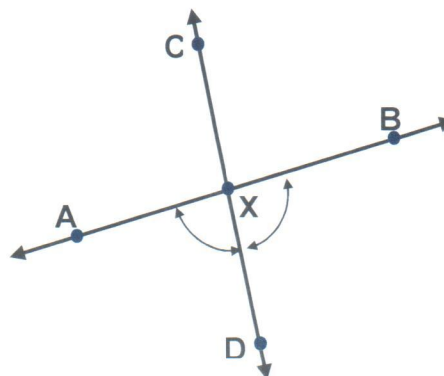
Example 4

Can you name more pairs of adjacent angles using the figure?

Solution:

$\angle AXD$ and $\angle DXB$

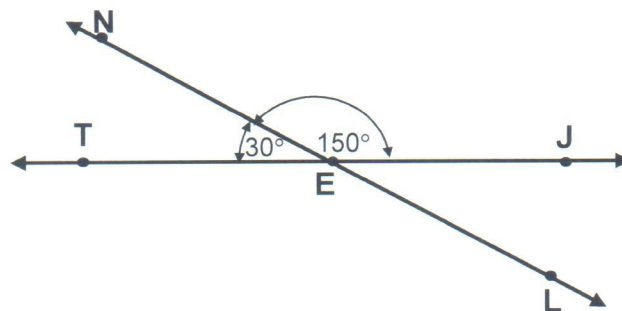
$\angle DXB$ and $\angle BXC$



Linear Pair Angles

Adjacent angles formed by two intersecting lines are always supplementary. They are also called **linear pair angles**. A **linear pair** refers to two angles formed by two intersecting lines which are **always supplementary and adjacent with each other**.

In the figure below, $\angle TEN$ and $\angle JEN$ form a linear pair. The two angles share a common side $\overline{EN}$ and a common vertex E . In addition the said angles are supplementary at the same time. Notice that $\angle TEN$ measures 30° while $\angle JEN$ measures 150° . Adding their measures, we have $30^\circ + 150^\circ = 180^\circ$. Since the two angles are adjacent and supplementary at the same time, therefore, they are considered as a linear pair.



Example 5

Using the same figure above, can you name more pairs of linear pair angles?

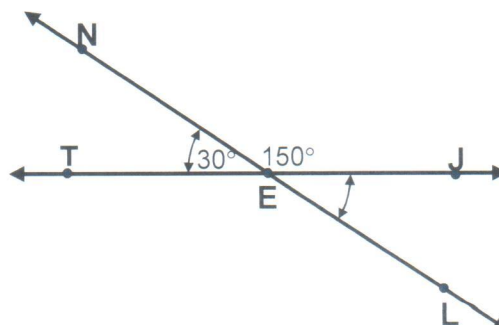
Solution:

$\angle NET$ and $\angle TEL$, $\angle TEL$ and $\angle JEL$, $\angle JEL$ and $\angle JEN$

Vertical Angles

Two angles formed by two intersecting lines which are opposites (non-adjacent) and share a common vertex are called **vertical angles**. These angles are **always congruent** with each other.

In the figure below, $\angle TEN$ and $\angle JEL$ are vertical angles. The two angles share a common vertex E . However, notice that the two angles are opposites or we could say that they are non-adjacent.



How do we prove that $\angle TEN$ is congruent with $\angle JEL$?

We have learned previously that $\angle \text{TEN}$ and $\angle \text{JEN}$ form a linear pair, thus, the sum of their measures is 180° . The same is true with $\angle \text{JEN}$ and $\angle \text{JEL}$. With this idea we can make an equation: ($m\angle \text{TEN}$ is read as “the measure of angle **TEN**”)

$$m\angle \text{TEN} + m\angle \text{JEN} = m\angle \text{JEN} + m\angle \text{JEL}$$

substituting their measures, we get

$$30^\circ + 150^\circ = 150^\circ + m\angle \text{JEL}$$

Adding the measure, we get

$$180^\circ = 150^\circ + m\angle \text{JEL}$$

Transposing 150° on the right side of the equation, we get

$$180^\circ - 150^\circ = m\angle \text{JEL}$$

Simplifying the equation

$$30^\circ = m\angle \text{JEL} \text{ or could also be written as } m\angle \text{JEL} = 30^\circ$$

Since the measure of $\angle \text{TEN}$ is 30° and the derived measure of $\angle \text{JEL}$ is also 30° , we therefore conclude that the two angles are congruent.

Can you now tell the measure of $\angle \text{TEL}$?

If your answer is 150° , then you are doing a great job!

NOW DO PRACTICE EXERCISE 1

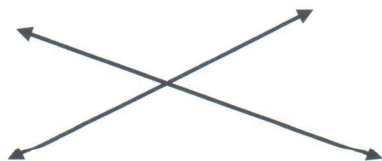
**Practice Exercise 1**

- 1) Write TRUE if the statement is correct; otherwise write FALSE on the blank space before each letter.

- _____ a) Vertical angles are congruent.
_____ b) Opposite angles are supplementary.
_____ c) Linear pair angles are supplementary.
_____ d) Two obtuse angles are supplementary.
_____ e) Parallel lines intersect at a common point called vertex.
_____ f) Two acute angles can be supplementary with each other.
_____ g) Adjacent angles share a common side and a common vertex.
_____ h) There are four linear pairs formed when two lines intersect each other.
_____ i) The measure of an angle which is supplementary with an angle measuring 80° is 10° .
_____ j) If the sum of two angles is exactly 90° , then the two angles are complementary.

- 2) Circle YES if the figure shows two intersecting lines. If the lines are not considered as intersecting lines based on the given definition then, circle NO and briefly explain why on the spaces provided.

a)



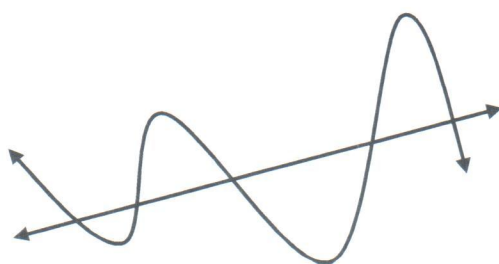
YES NO

b)



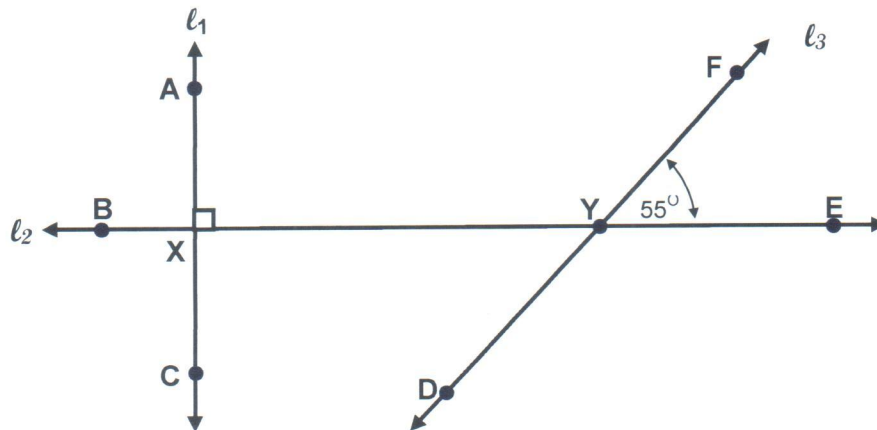
YES NO

c)



YES NO

- 3) Use the given figure and information below to answer the following questions. Write the answers on the blank spaces provided.



- a) Which line intersects line 2 at point Y? _____
- b) Which angles are adjacent with $\angle AXB$? _____ and _____
- c) If the measure of $\angle FYE$ is 55° , what are the measures of the following?
- $\angle EYD$ _____
 - $\angle FYB$ _____
 - $\angle DYB$ _____
- d) Name two pairs of vertical angles formed by the intersection of Line 1 and 2.
 _____ and _____
- e) Name four pairs of adjacent angles formed by the intersection of Line 2 and 3.

- f) What relationship exists between the following angles? Write None if there is no found relationship.
- $\angle AXB$ and $\angle EXC$ _____
 - $\angle AXB$ and $\angle BXC$ _____
 - $\angle AXB$ and $\angle AXE$ _____
 - $\angle AXB$ and $\angle EYD$ _____
 - $\angle EYD$ and $\angle FYB$ _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 2: Angle Properties of Parallel Lines



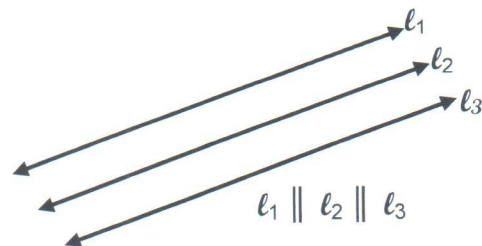
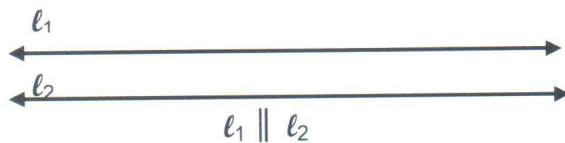
You have learnt about angle properties of intersecting lines in Lesson 1.



In this lesson, you will:

- define a transversal line
- identify the angle properties associated with parallel lines cut by a transversal
- solve problems involving angles formed by parallel lines and its transversal.

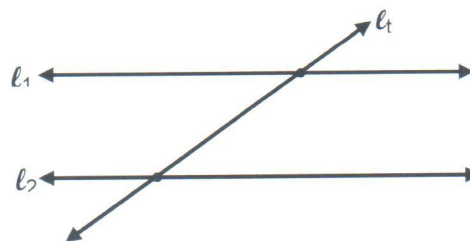
You have discussed in your previous Maths that two or more lines that do not meet or intersect and are equidistant (equal distance) from each other are called parallel lines. The figures below show the idea of what parallel lines are.



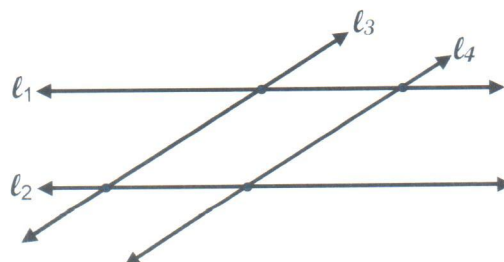
Transversal Line

A line which intersects two or more parallel lines is known as a **transversal line**. Usually a transversal line is denoted by the symbol l_t .

In the figure below $l_1 \parallel l_2$ these lines are intersected by the transversal line l_t at two different points.



In the figure below, $l_1 \parallel l_2$ and $l_3 \parallel l_4$. Observe that both l_1 and l_2 are cut by l_3 and l_4 at different points. With that we can say that l_3 and l_4 are the transversals of the parallel lines l_1 and l_2 .



Example 1

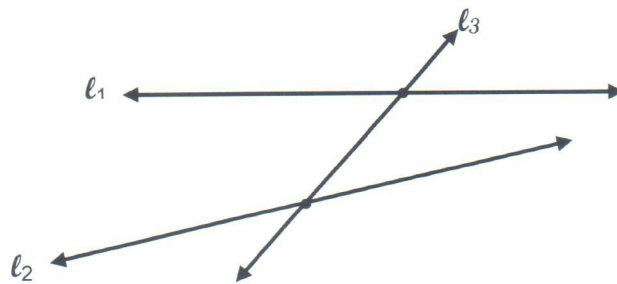
Can we consider ℓ_1 and ℓ_2 as transversal lines too?

Solution:

Yes. Because these two lines intersect the parallel lines ℓ_3 and ℓ_4 .

Example 2

Is line 3 in the next figure a transversal line?



Solution:

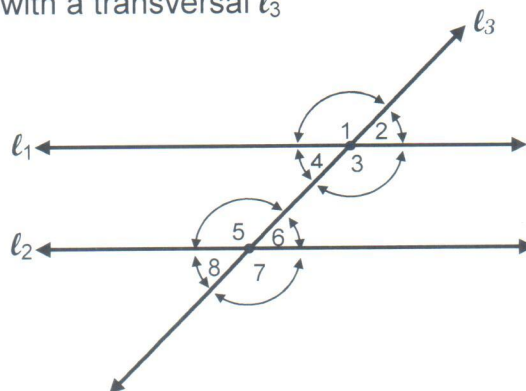
No, ℓ_3 could not be considered as a transversal line. Although it intersects ℓ_1 and ℓ_2 at two different points still we could not consider it as such because it is very obvious that ℓ_1 and ℓ_2 are not parallel lines.

Angles Formed with Transversal Line

It is very interesting to note that when parallel lines are cut by a transversal, angle relationships with their unique properties are formed.

Observe the figure below:

Given: $\ell_1 \parallel \ell_2$ with a transversal ℓ_3



In lesson 1, we discussed linear pair angles and vertical angles. Can you name pairs of vertical angles formed?

Answers:

$\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 5$ and $\angle 7$, $\angle 6$ and $\angle 8$

How about linear pair angles, can you name them?

Answers:

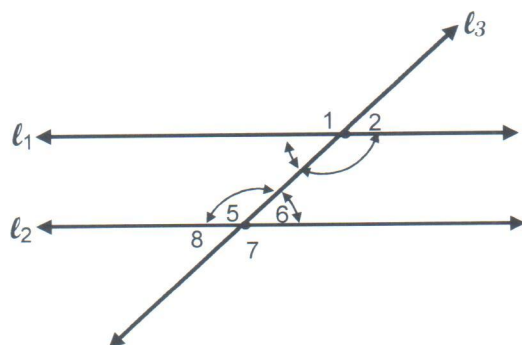
$\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 1$ and $\angle 4$
 $\angle 5$ and $\angle 6$, $\angle 6$ and $\angle 7$, $\angle 7$ and $\angle 8$, $\angle 5$ and $\angle 8$

Now, do not forget the properties of these angle pairs because it will also help us later discover how the other angles relate with each other.

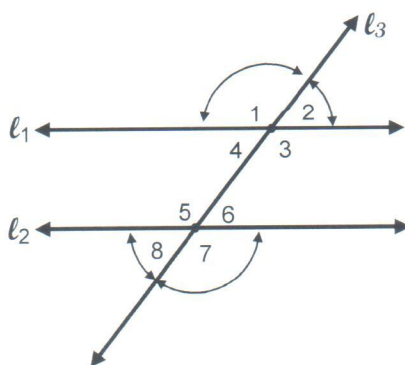
Interior Angles

Interior angles (co-interior angles) are formed when parallel lines are cut by a transversal. These angles lie in between two parallel lines.

In the figure below $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are interior angles. They lie in between the parallel lines l_1 and l_2 .



How about $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$?



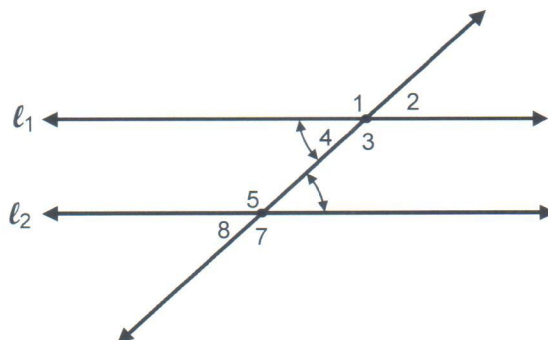
These angles are called **exterior angles** since they do not lie in between the parallel lines.

Alternate Angles

Alternate angles are formed when parallel lines are cut by a transversal. These angles lie on opposite sides of the transversal line. Both must either be in the interior (Alternate Interior Angles) or on the exterior (Alternate Exterior Angles) sides of the parallel lines.

In the figure below, $\angle 4$ and $\angle 6$ are alternate interior angles. Since one of their properties states that alternate interior angles are congruent, we can now say that $\angle 4 \cong \angle 6$ (read as $\angle 4$ is congruent with $\angle 6$).

Congruent means same size and shape. Two angles are congruent, if they have the same measure.



Example 3

What are the other pairs of alternate angles in the figure?

Solution:

$\angle 3$ and $\angle 5$ are another pair of alternate interior angles.

$\angle 1$ and $\angle 7$ are pairs of
 $\angle 2$ and $\angle 8$ alternate exterior angles

ALTERNATE INTERIOR ANGLES THEOREM

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

In the figure above, we said that $\angle 4$ and $\angle 6$ are alternate interior angles. Since the ALTERNATE INTERIOR ANGLES THEOREM states that alternate interior angles are congruent, we can now say that $\angle 4 \cong \angle 6$ (read as $\angle 4$ is congruent with $\angle 6$).

Suppose we assign 50° as the measure of $\angle 4$, then, we can directly conclude that $\angle 6$ also measures 50° because of the theorem.

Example 4

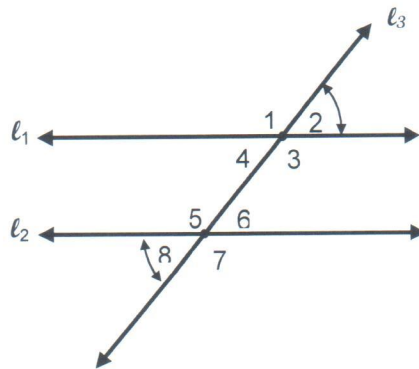
What are the measures of $\angle 3$ and $\angle 5$?

Solution:

Since $\angle 3$ and $\angle 4$ are supplementary angles, we say $m\angle 3 + 50^\circ = 180^\circ$, then $m\angle 3 = 180^\circ - 50^\circ = 130^\circ$. Knowing that $\angle 3$ and $\angle 5$ are congruent, and then we say $\angle 5$ also measures 130° .

ALTERNATE EXTERIOR ANGLES THEOREM

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



In the figure above, we said that $\angle 2$ and $\angle 8$ are alternate exterior angles. Since the **ALTERNATE EXTERIOR ANGLES THEOREM** states that alternate exterior angles are congruent, we can now say that $\angle 2 \cong \angle 8$.

Suppose we assign 120° as the measure of $\angle 2$, then, we can directly conclude that $\angle 8$ also measures 120° because of the theorem.

Example 5

What are the measures of $\angle 1$ and $\angle 7$?

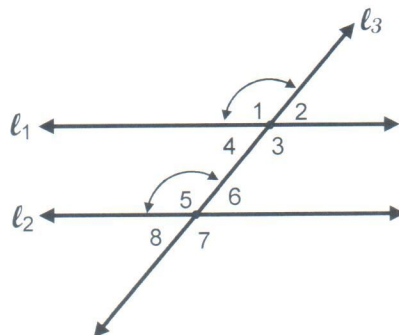
Solution:

Since $\angle 1$ and $\angle 2$ are supplementary angles, we say $m\angle 1 + 120^\circ = 180^\circ$, then, $m\angle 1 = 180^\circ - 120^\circ = 60^\circ$. Knowing that $\angle 1$ and $\angle 7$ are congruent then we say $\angle 7$ also measures 60° .

Corresponding Angles

Corresponding angles are formed when parallel lines are cut by a transversal. Specifically these angles lie on the same side of the transversal however, one is an interior and the other is an exterior angle.

The figure below shows that, $l_1 \parallel l_2$ with a transversal l_3 .



In the figure, $\angle 1$ and $\angle 5$ are corresponding angles. $\angle 1$ is an exterior angle while $\angle 5$ is an interior angle.

Notice that both angles are on the left side of the transversal.

Example 6

Name more pairs of corresponding angles.

Solution:

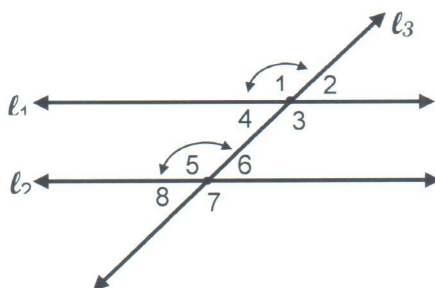
$\angle 2$ and $\angle 6$

$\angle 4$ and $\angle 8$

$\angle 3$ and $\angle 7$

CORRESPONDING ANGLES POSTULATE

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



In the figure above, we said that $\angle 1$ and $\angle 5$ are corresponding angles. Since the CORRESPONDING ANGLES THEOREM states that corresponding angles are congruent, we can now say that $\angle 1 \cong \angle 5$.

Suppose we assign 100° as the measure of $\angle 1$, then, we can directly conclude that $\angle 5$ also measures 100° because of the theorem.

Example 7

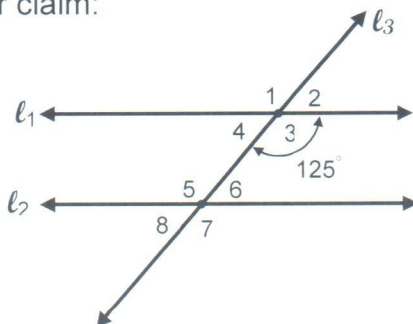
What are the measures of $\angle 2$ and $\angle 6$?

Solution:

Since $\angle 1$ and $\angle 2$ are supplementary angles, we say $m\angle 2 + 100^\circ = 180^\circ$, then, $m\angle 2 = 180^\circ - 100^\circ = 80^\circ$. Knowing that $\angle 2$ and $\angle 6$ are congruent, and then we say $\angle 6$ also measures 80° .

Example 8

Give the measure of the following angles in relation to $\angle 3 = 125^\circ$ and state the reason(s) that justifies your claim:



a) $\angle 1$

b) $\angle 2$

c) $\angle 4$

d) $\angle 5$

e) $\angle 7$

f) $\angle 6$

g) $\angle 8$

Solution:

Given	Measure	Reason(s)
a) $\angle 1$	125°	$\angle 1 \cong \angle 3$ (vertical angles are congruent)
b) $\angle 2$	55°	$\angle 2$ and $\angle 3$ forms a linear pair, therefore they are supplementary. $m\angle 2 + m\angle 3 = 180^\circ$ $m\angle 2 = 180^\circ - 125^\circ = 55^\circ$
c) $\angle 4$	55°	$\angle 4$ and $\angle 3$ forms a linear pair, therefore they are supplementary. $m\angle 4 + m\angle 3 = 180^\circ$ $m\angle 4 = 180^\circ - 125^\circ = 55^\circ$
d) $\angle 5$	125°	$\angle 5 \cong \angle 3$ (Refer to Alternate Interior Angles Postulate)
e) $\angle 7$	125°	$\angle 7 \cong \angle 3$ (Refer to Corresponding Angles Postulate)
f) $\angle 6$	55°	Although $\angle 6$ is not directly related with $\angle 3$, we can say that its measure is congruent with $\angle 7$ (measure derived as stated above in solution e. $\angle 6 \cong \angle 7$ (vertical angles are congruent)
g) $\angle 8$	55°	Although $\angle 8$ is not directly related with $\angle 3$, we can say that its measure can be derived in relation with $\angle 7$. $\angle 8$ and $\angle 7$ form a linear pair, therefore they are supplementary. $m\angle 8 + m\angle 7 = 180^\circ$ $m\angle 8 = 180^\circ - 125^\circ = 55^\circ$

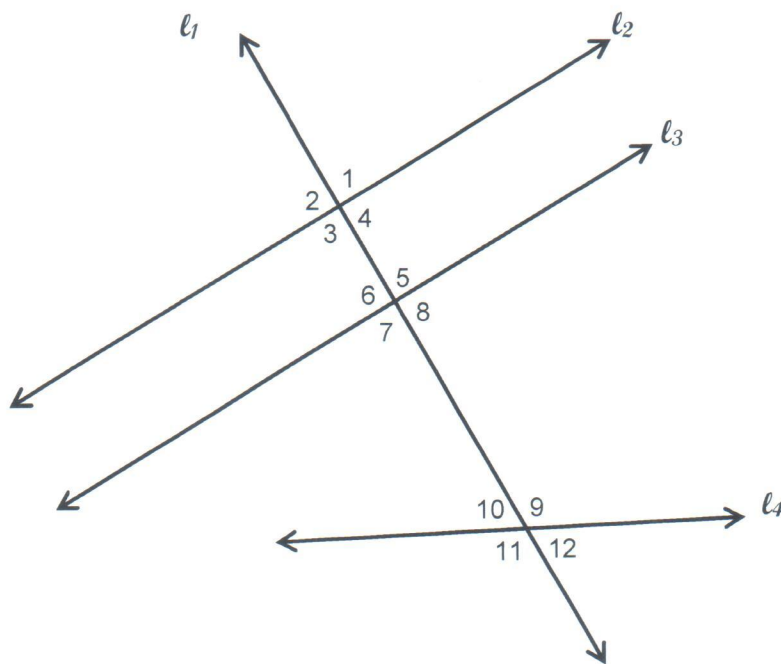
Now you know the relationships of different angles formed by parallel lines cut by a transversal. Review Lesson 2 and prepare to answer the next Practice Exercise.

NOW DO PRACTICE EXERCISE 2



Practice Exercise 2

1. Use the figure below to answer the following questions.



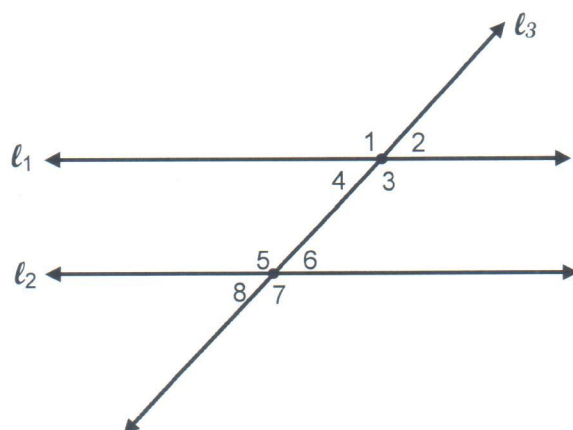
- a) Name a pair of parallel lines. _____
- b) Name a transversal line. _____
- c) What line intersects line 4? _____
- d) Name 2 pairs of vertical angles formed by line 1 and line 4.

- e) Name all interior angles in the figure.

- f) Name all exterior angles in the figure.

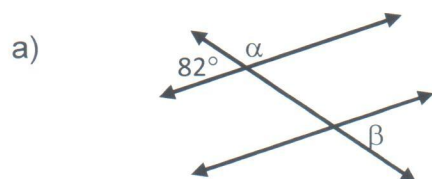
- g) Name the angle relationship between the given angles.
- i. $\angle 1$ and $\angle 2$ _____
 - ii. $\angle 3$ and $\angle 5$ _____
 - iii. $\angle 1$ and $\angle 7$ _____
 - iv. $\angle 5$ and $\angle 7$ _____
 - v. $\angle 1$ and $\angle 4$ _____
 - vi. $\angle 2$ and $\angle 6$ _____
 - vii. $\angle 9$ and $\angle 12$ _____

2. Use the figure below to answer the following questions:

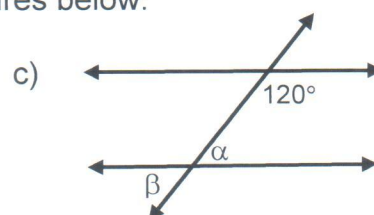


- a) What is the measure of $\angle 2$ if $\angle 1 = 125^\circ$? Answer: _____
- b) What is the measure of $\angle 4$ if $\angle 1 = 125^\circ$? Answer: _____
- c) What is the measure of $\angle 5$ if $\angle 7 = 140^\circ$? Answer: _____
- d) What is the measure of $\angle 8$ if $\angle 2 = 45^\circ$? Answer: _____
- e) What is the measure of $\angle 3$ if $\angle 7 = 150^\circ$? Answer: _____

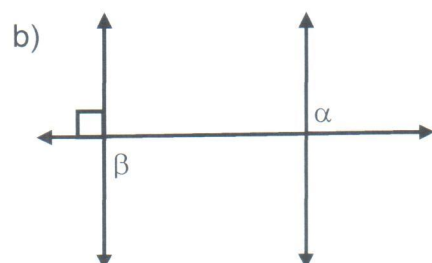
3. Find the value of the pronumerals shown in the figures below:



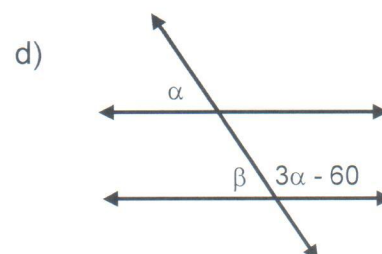
Answer: $\alpha =$ _____
 $\beta =$ _____



Answer: $\alpha =$ _____
 $\beta =$ _____



Answer: $\alpha =$ _____
 $\beta =$ _____



Answer: $\alpha =$ _____
 $\beta =$ _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 3: Angle Properties of Triangles



You have learned about Angle Properties of Parallel Lines in the previous lesson.



In this lesson you will:

- revise properties of different triangles
- classify triangles defined by sides and angles they contain
- discuss ways to solve angle problems concerning triangles.

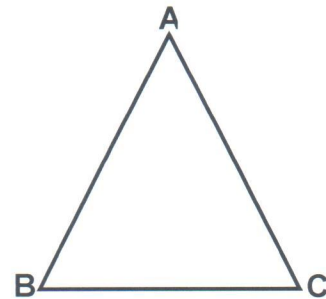
In your previous Mathematics lessons, you learnt that **a triangle is a polygon with three sides**. In addition, you also learnt that **a triangle has three endpoints** or known as vertices.

In this lesson we will explore more about triangles and its properties.

The figure at the right is a triangle. The triangle can be named as triangle **ABC** or using the symbol **$\triangle ABC$** (read as triangle ABC).

The triangle has three sides. These are $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$.

It also contains three angles namely $\angle A$, $\angle B$ and $\angle C$.



Classification of Triangles

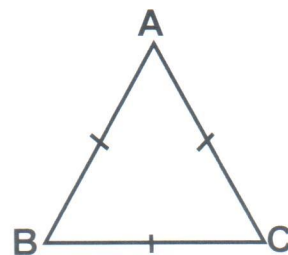
Triangles are classified according to their sides and angles they contain. Each classification holds special properties.

- **defined by sides**

Equilateral Triangle is a triangle whose sides are equal.

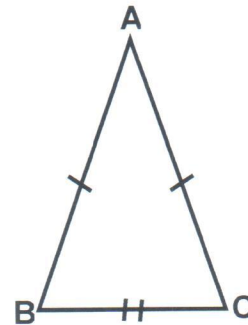
Given $\triangle ABC$, it is an equilateral triangle because $\overline{AB} = \overline{BC} = \overline{AC}$. This means that each side of the triangle has the same measure.

The marker ($\simeq$) indicates equality.



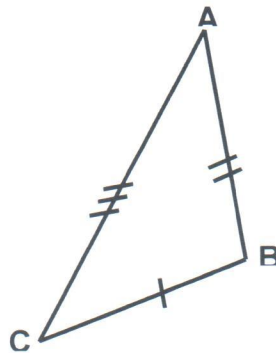
Isosceles Triangle is a triangle wherein two (2) sides are equal.

Given $\triangle ABC$, it is an isosceles triangle because $\overline{AB} = \overline{AC}$. This means that two sides of the triangle have the same measure.



Scalene Triangle is a triangle with no equal sides.

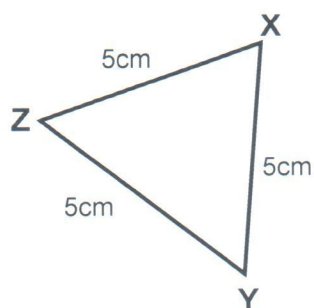
Given $\triangle ABC$, it is a scalene triangle because $\overline{AB} \neq \overline{BC} \neq \overline{AC}$. This means none of the sides of the triangle have the same measure. Each side of the triangle has a unique measure different from the others.



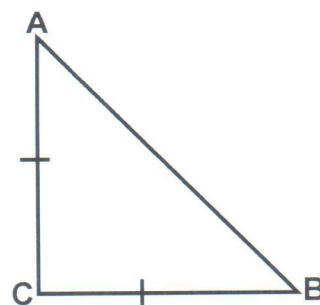
Example 1

Classify the following triangles defined by their sides.

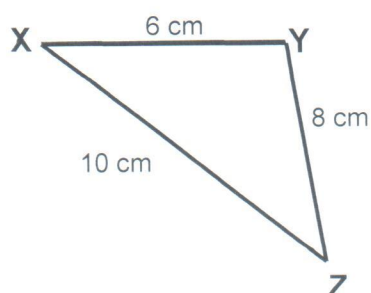
a)



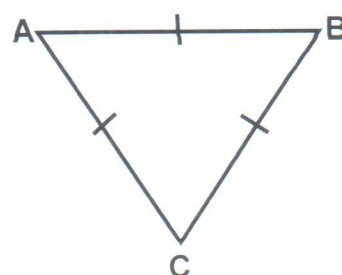
b)



c)



d)



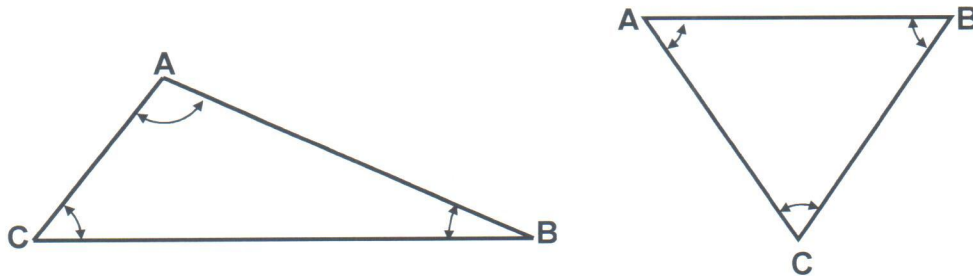
Solution:

- It is an **equilateral triangle**. Since $\overline{XY} = \overline{YZ} = \overline{XZ}$. Each side measures 5 cm.
- It is an **isosceles triangle**. Since $\overline{AC} = \overline{BC}$. Two sides of the triangle have the same measure.
- It is a **scalene triangle**. Since $\overline{XY} \neq \overline{YZ} \neq \overline{XZ}$. Each sides of the triangle have a unique measure. No sides are congruent with each other.
- It is an **equilateral triangle**. Since $\overline{AB} = \overline{BC} = \overline{AC}$. Each side measures the same as labelled in the figure in (d).

- defined by angles**

Acute Triangle is a triangle which contains three (3) acute angles. All of its angles measures less than 90° .

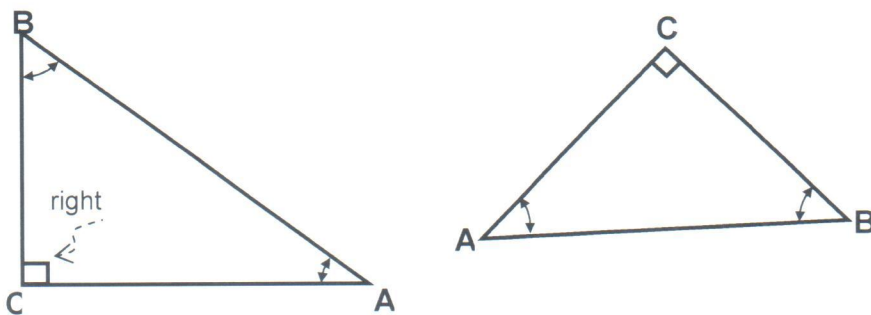
The following are examples of acute angle.



Notice that in each triangle, $\angle A$, $\angle B$ and $\angle C$ are all acute angles.

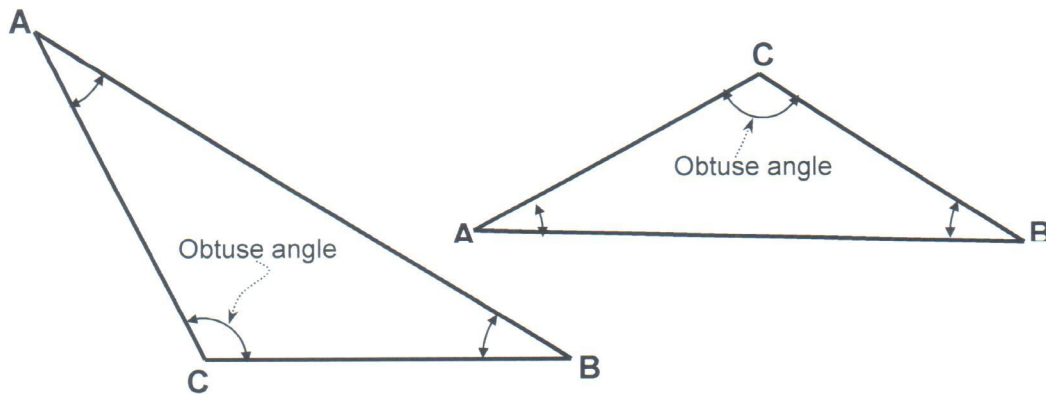
Right triangle (or Right-angled triangle) is a triangle which contains one (1) right angle and two (2) acute angles. This triangle has one 90° angle.

The following are examples of right triangle.



Notice that in each triangle $\angle C$ is a right angle while $\angle B$ and $\angle A$ are acute angles.

Obtuse Triangle is a triangle which contains one (1) obtuse angle and two (2) acute angles. This triangle has one angle which measures more than 90° .



Notice that in each triangle $\angle C$ is an obtuse angle while $\angle A$ and $\angle B$ are acute angles.

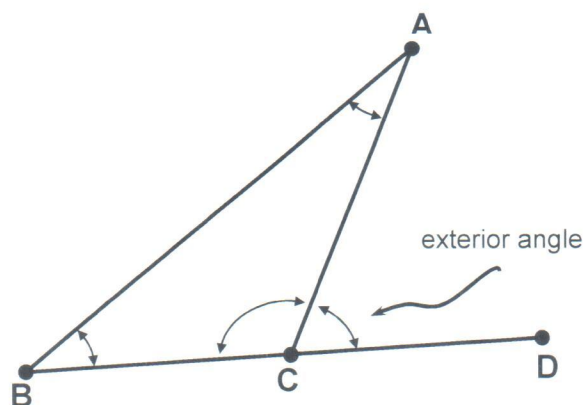
Angle Properties of Triangles

Now that you already know how to classify triangles according to their sides and angles they contain, it is important to note that all these triangles share some common properties. It means that the following properties hold true for each of the triangles classified previously.

Triangle Angle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180° .

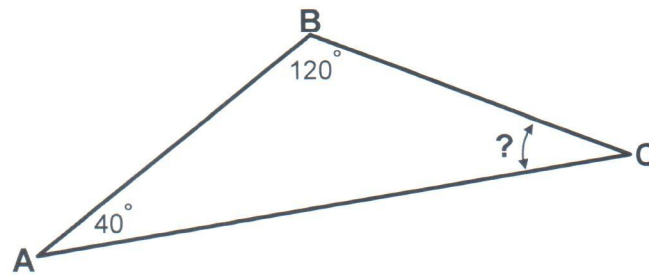
In the figure below $\angle A$, $\angle B$ and $\angle C$ are the interior angles of $\triangle ABC$ while $\angle ACD$ is an exterior angle.



The Triangle Sum Theorem means that $m\angle A + m\angle B + m\angle C = 180^\circ$.

Example 2

Find the measure of $\angle C$ in the figure.



Solution:

Since we know that $m\angle A + m\angle B + m\angle C = 180^\circ$, we will substitute the given values in the equation to solve for the measure of $\angle C$.

Triangle Sum Theorem

$$m\angle A + m\angle B + m\angle B = 180^\circ$$

Substituting the given values

$$40^\circ + 120^\circ + m\angle C = 180^\circ$$

Add 40° and 120°

$$160^\circ + m\angle C = 180^\circ$$

Transpose 160° on the right side

$$m\angle C = 180^\circ - 160^\circ$$

Get the difference

$$m\angle C = 20^\circ$$

Therefore, the measure of angle C is 20° .

Example 3

What is the measure of the 3rd angle of a triangle whose two angles measures 30° and 70° respectively?

Solution:

In this problem the triangle is not drawn but the measures of its two angles are given. These given information are enough to help us solve for the measure of the third side. According to the Triangle Sum Theorem, The sum of the three angles of the triangle is equal to 180° . Having this in mind, we can now say that 1st angle + 2nd angle + 3rd angle = 180° .

$$1^{\text{st}} \text{ angle} + 2^{\text{nd}} \text{ angle} + 3^{\text{rd}} \text{ angle} = 180^\circ$$

$$30^\circ + 70^\circ + 3^{\text{rd}} \text{ angle} = 180^\circ$$

$$100^\circ + 3^{\text{rd}} \text{ angle} = 180^\circ$$

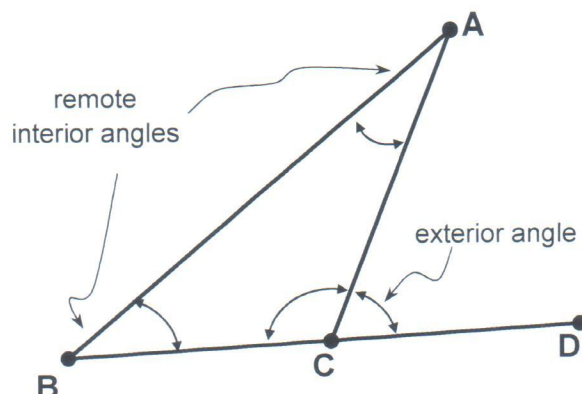
$$3^{\text{rd}} \text{ angle} = 180^\circ - 100^\circ$$

$$3^{\text{rd}} \text{ angle} = 80^\circ$$

Therefore, the measure of the 3rd angle is 80° .

Exterior Angle Theorem

The next theorem involves the remote interior angles of a triangle. The two angles that are not adjacent or next to the exterior angle of a triangle are called **remote interior angles**. The figure below shows that $\angle A$ and $\angle B$ are the remote interior angles while $\angle ACD$ is the exterior angle.



These terms are very important to understand clearly the Exterior Angle Theorem.

Exterior Angle Theorem

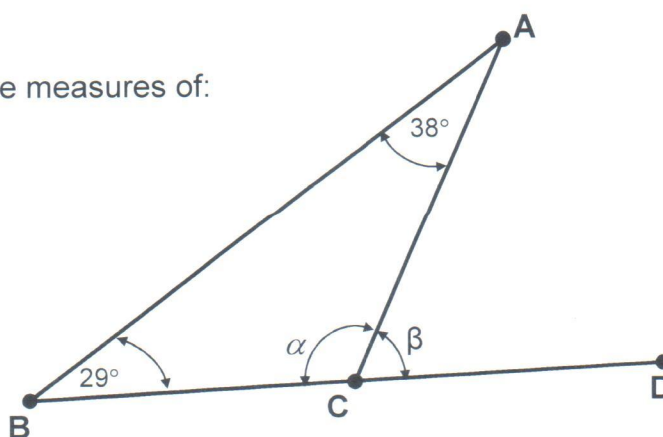
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

The Exterior Angle Theorem means that $m\angle A + m\angle B = m\angle ACD$.

Example 4

Using the figure below find the measures of:

- $\angle ACB$
- $\angle ACD$



Solution:

- To solve for the measure of $\angle ACB$, we will use the Triangle Angle Sum Theorem.

Triangle Sum Theorem

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

Substituting the given values

$$38^\circ + 29^\circ + m\angle C = 180^\circ$$

Add 38° and 29°

$$67^\circ + m\angle C = 180^\circ$$

Transpose 67° on the right side

$$m\angle C = 180^\circ - 67^\circ$$

Get the difference

$$m\angle C = 113^\circ$$

- b) To solve for the measure of $\angle ACB$, we will use the Exterior Angle Theorem

Exterior Angle Theorem

$$m\angle A + m\angle B = m\angle ACD$$

Substituting the given values

$$38^\circ + 29^\circ = m\angle ACD$$

Add $38^\circ + 29^\circ$

$$67^\circ = m\angle ACD$$

Therefore the measure of $\angle ACB$ or also named as $\angle C$ is 113° , while the measure of $\angle ACD$ is 67° .

To check if the answers are correct, add their measures.

$$113^\circ + 67^\circ = 180^\circ$$

$$180^\circ = 180^\circ$$

Since the sum is 180° , then the answers are correct.

Why 180° ?

Simply because the figure shows that the two angles form a line. As we discussed in our previous lessons, these angles are supplementary.

Did you think it that way?

Great!

Now you are ready for the next Practice Exercise.

NOW DO PRACTICE EXERCISE 3



Practice Exercise 3

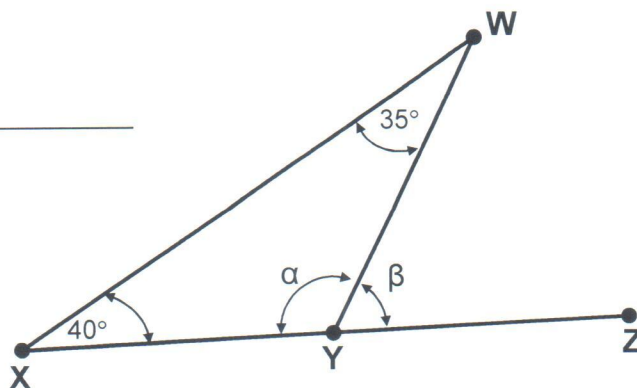
1. Classify the following triangles according to their sides and angles contained. Write your answers on the table below. The first one is done for you.

Given Triangle	Classification defined by	
	length of sides	angles contained
	Equilateral Triangle	Acute triangle
a)		
b)		
c)		
d)		
e)		

2. Find the measures of the following angles using the figure below:

a) $\angle WYX$

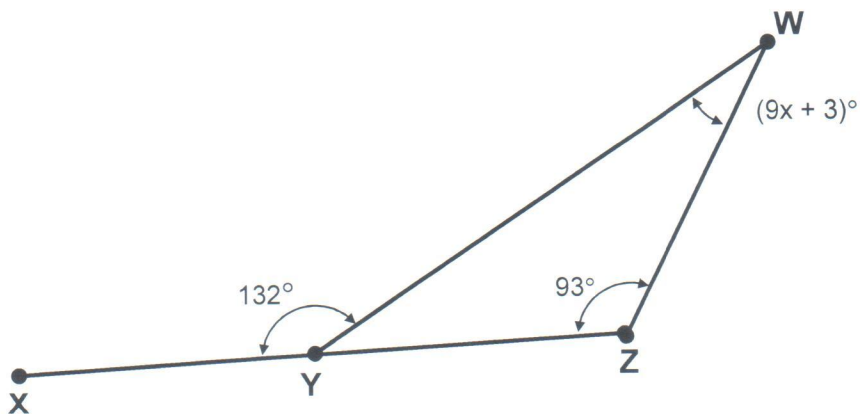
Answer: _____



b) $\angle WYZ$

Answer: _____

3. Find the measure of the following angles using the figure below:



a) $\angle YWZ$

Answer: _____

b) $\angle WYZ$

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 4: Similar Triangles



You learnt something triangles and their classifications in Lesson 3.



In this lesson you will:

- revise and discuss the properties of similar triangles
- enumerate and discuss ways to find the lengths of sides of similar triangles
- solve problems involving similar triangles.

What does similarity mean?



A shirt shop went on a sale. Its sign board says “Buy 2 Similar Shirts for the price of 1”. Now you see the 3 shirts above on display. Which shirts will you pick?

The word “similar” is sometimes synonymously used as the word “same”. Others confusingly use it to replace the word “congruent”. These words may be synonymous in a way in written English but in Mathematics they really mean different. So if you say that Shirt 2 and Shirt 3 are similar because their sizes are Large, then you mean the two are congruent instead of similar. Having exactly the same size refers to the idea of congruency.

What about similarity?

Yes you are correct if you say Shirt 1 and Shirt 2 are similar. Although one is Small in size while the other is Large in size, they are still similar because they are exactly the same (in prints, colour and type). They are both short-sleeved unlike the third shirt which is a long-sleeved. As the size of the shirt grows bigger, all the dimensions of the shirt such as its width, length, collar length, shoulder length and width grow bigger too in proportion.

Similar Triangles

The concept of similarity may also be applied to triangles. This knowledge will be of great help in solving real-life problems.

Two triangles are similar if their

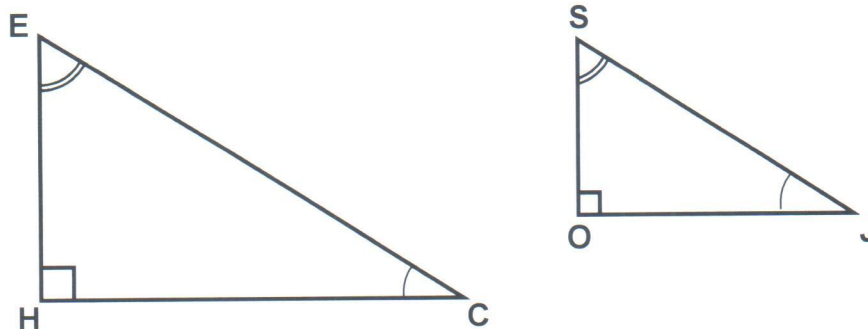
- **corresponding or matching angles are congruent or they have equal measures**
- **corresponding sides are in proportion**

The symbol for similarity is “ $\sim$ ”.

In Lesson 3 we discussed classifications of triangles. How could this knowledge help us deal with similar triangles?

Remember, we said that similar triangles must have congruent angles. Therefore, we cannot consider a right triangle to be similar with an obtuse triangle. Why? Obviously, a right triangle contains a 90-degree angle while an obtuse triangle has an angle more than 90°. With that, we can only work on similar triangles if they have the same classification. But there's more, the sides must form a proportion before we could conclude that two triangles are similar.

Let us observe the following triangles



Given $\triangle CHE$ and $\triangle JOS$, $\angle C$ corresponds to $\angle J$, $\angle H$ (read as right angle H) corresponds to $\angle O$ and $\angle E$ corresponds to $\angle S$. These are the three pairs of corresponding angles of the two triangles. $\overline{EH}$ corresponds to $\overline{SO}$, $\overline{EC}$ corresponds to $\overline{SJ}$ and $\overline{HC}$ corresponds to $\overline{OJ}$. These are the three pairs of corresponding sides of the two triangles.

Following the definition of what similar triangles are, we can say that $\triangle CHE \sim \triangle JOS$ (read as triangle **CHE** is similar to triangle **JOS**) if

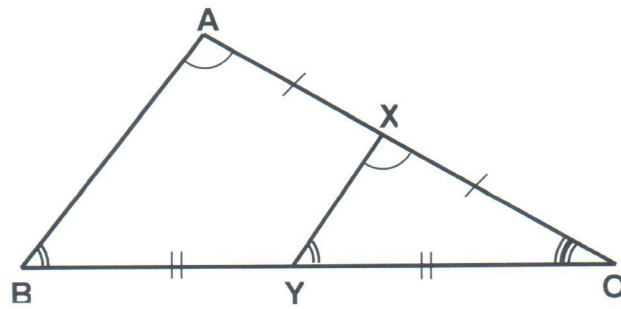
three sets of corresponding angles are congruent:

$$\angle C \cong \angle J, \angle H \cong \angle O \text{ and } \angle E \cong \angle S.$$

Corresponding sides are proportional:

$$\frac{\overline{EH}}{\overline{SO}} = \frac{\overline{EC}}{\overline{SJ}} = \frac{\overline{HC}}{\overline{OJ}} \quad \text{or can also be written as} \quad \frac{\overline{SO}}{\overline{EH}} = \frac{\overline{SJ}}{\overline{EC}} = \frac{\overline{OJ}}{\overline{HC}}$$

Sometimes the triangles are overlapping with each other like the figure below.



The figure shows two triangles namely ΔAOB (bigger triangle) and ΔXOY (smaller triangle). These triangles are congruent because their corresponding angles are congruent. These are $\angle A \cong \angle X$, $\angle B \cong \angle Y$ and $\angle O \cong \angle O$. $\angle O$ is congruent to itself based on reflexive property of congruency. $\angle O$ is contained in both triangles that is why it is congruent to itself.

Their corresponding sides are also proportional

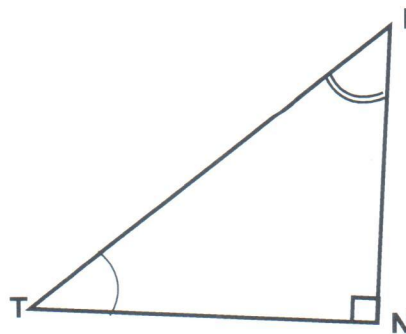
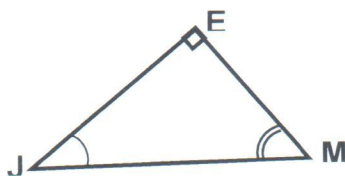
$$\frac{\overline{AB}}{\overline{XY}} = \frac{\overline{AO}}{\overline{XO}} = \frac{\overline{BO}}{\overline{YO}} \quad \text{or can also be written as} \quad \frac{\overline{XY}}{\overline{AB}} = \frac{\overline{XO}}{\overline{AO}} = \frac{\overline{YO}}{\overline{BO}}$$

When forming proportion using the sides of the triangles, take note that if you used the measures of the bigger triangle first, use it as numerator throughout the proportion or vice versa.

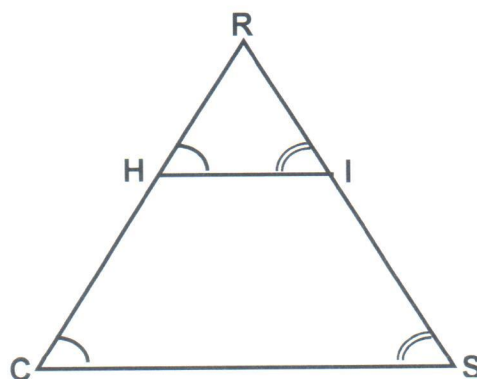
Example 1

Name the pairs of corresponding angles and sides given the following similar triangles.

a)



b)



Solution:

pairs of corresponding angles	pairs of corresponding sides
a) $\angle J$ and $\angle T$ $\angle E$ and $\angle N$ $\angle M$ and $\angle I$	$\overline{JE}$ and $\overline{TN}$ $\overline{EM}$ and $\overline{NI}$ $\overline{JM}$ and $\overline{TI}$
b) $\angle C$ and $\angle H$ $\angle S$ and $\angle I$ $\angle R$ and $\angle R$	$\overline{RC}$ and $\overline{RH}$ $\overline{RS}$ and $\overline{RI}$ $\overline{CS}$ and $\overline{HI}$

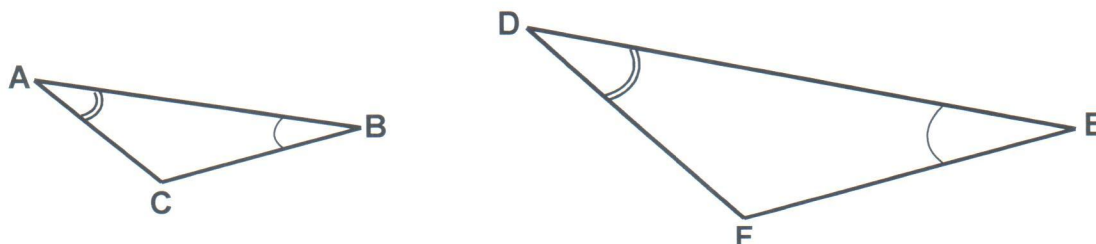
Sometimes we need not to prove all corresponding angles are congruent and all sides form a proportion to conclude that the triangles are similar. The following theorems will help us tell triangle similarity by just observing some of the corresponding parts of the triangles.

Properties of Similar Triangles

AA Similarity Theorem

If two angles of one triangle are congruent to their corresponding angles of another triangle, then the two triangles are similar.

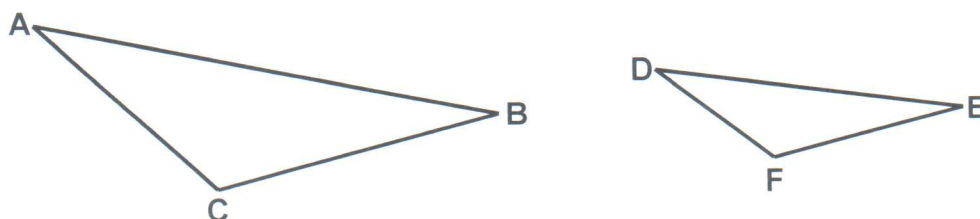
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.



SSS (Side-Side-Side) Similarity Theorem

If the three pairs of corresponding sides of two triangles are in proportion, then the two triangles are similar.

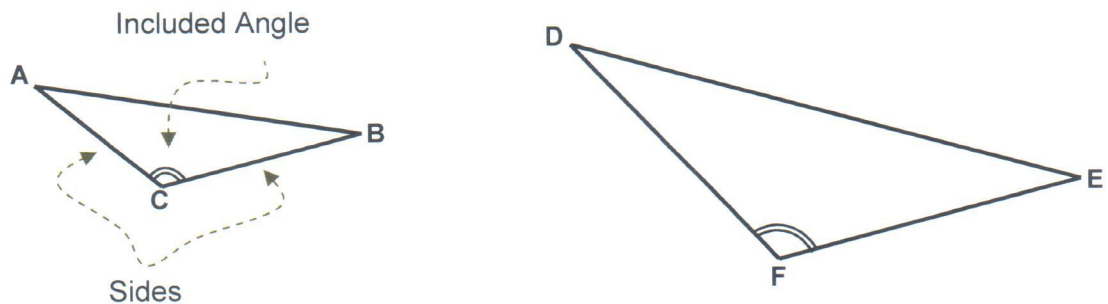
If $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ (can also be written as $\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$), then $\triangle ABC \sim \triangle DEF$.



SAS (Side-Angle-Side) Similarity Theorem

If the two pairs of corresponding sides of two triangles are in proportion and their included angle are congruent then the two triangles are similar.

If $\frac{AC}{DF} = \frac{BC}{EF}$ and $\angle C \cong \angle F$, then $\triangle ABC \sim \triangle DEF$.

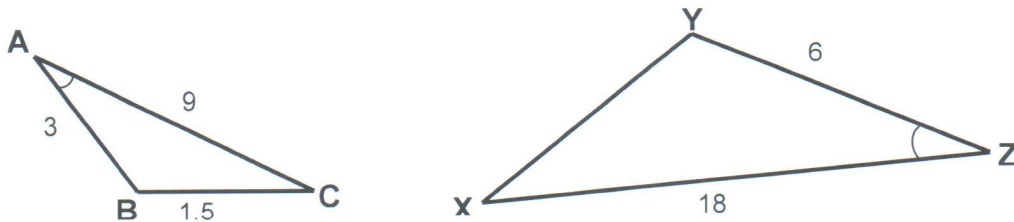


Note: Do not be confused with the SAS and SSS theorems for congruency. They are different from SAS and SSS Similarity Theorems because the sides involved in similarity theorems involve proportions among them.

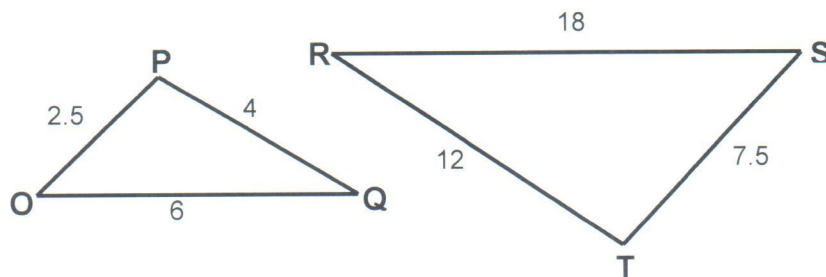
Example 2

Name the Similarity Theorem that justifies the similarity of the given triangles.

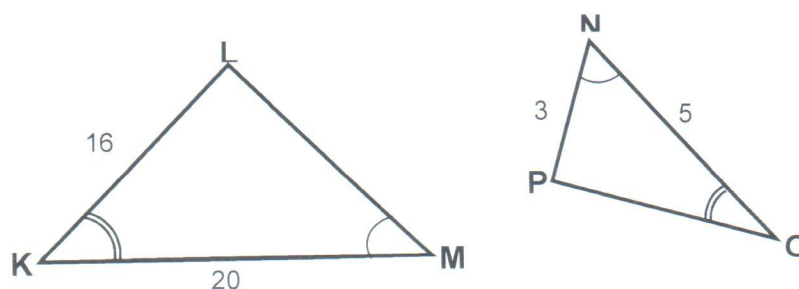
a)



b)



c)



Solution:

a) SAS Similarity Theorem.

The two pairs of corresponding sides $\frac{AC}{ZX} = \frac{AB}{ZY}$ are in proportion and their included angle $\angle A \cong \angle Z$. To verify the proportionality of the sides, substitute the measures of each side $\frac{9}{18} = \frac{3}{6}$. Simplifying both sides of the equation $\frac{1}{2} = \frac{1}{2}$.

b) SSS Similarity Theorem.

The three pairs of corresponding sides $\frac{OP}{ST} = \frac{OQ}{SR} = \frac{PQ}{TR}$ are in proportion. To verify the proportionality of the sides $\frac{2.5}{7.5} = \frac{6}{18} = \frac{4}{12}$. Simplifying all ratios we get $\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$.

c) AA Similarity Theorem.

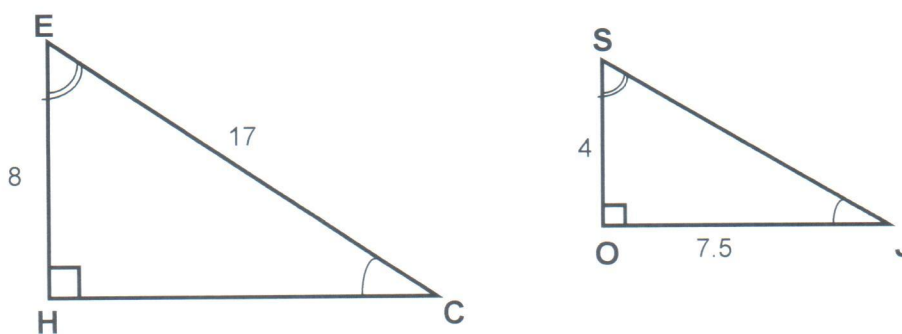
Although there are measures of some sides labelled, a proportion cannot be formed. But the two triangles are still similar because the two pairs of corresponding angles are congruent. These are $\angle K \cong \angle O$ and $\angle M \cong \angle N$.

Now that you already mastered the ways to identify the properties or theorems that justify triangle congruency, it will be very easy for you to learn how to solve for the measure of the missing side of a triangle using triangle similarity.

Finding the Missing Part(s) of Similar Triangles

Knowing that two triangles are similar, one can solve for the missing parts of the triangles by forming a proportion using any two pairs of corresponding sides of the triangle.

Let us observe the following triangles



Given $\triangle CHE \sim \triangle JOS$, through the AA Similarity theorem because at least two of the three sets of corresponding angles are congruent namely $\angle C \cong \angle J$, $\angle H \cong \angle O$ and $\angle E \cong \angle S$.

A proportion can be formed through the pairs of corresponding sides

$$\frac{\overline{EH}}{\overline{SO}} = \frac{\overline{EC}}{\overline{SJ}} = \frac{\overline{HC}}{\overline{OJ}} \text{ or can also be written as } \frac{\overline{SO}}{\overline{EH}} = \frac{\overline{SJ}}{\overline{EC}} = \frac{\overline{OJ}}{\overline{HC}}$$

In the figures on the previous page, each triangle has a missing part. $\overline{HC}$ for $\triangle CHE$ and $\overline{SJ}$ for $\triangle JOS$. Observing the figures above, we can form a proportion using two pairs of corresponding sides. One pair must contain the unknown side and the other pair must have values on both numerator and denominator.

Example 3

Solve for the measure of HC in the figures above.

Solution:

We notice that EH corresponds to SH and both have values. So we can form the ratio $\frac{8}{4}$. Since the unknown is HC, we will use HC with its corresponding side OJ.

With this we can form the ratio $\frac{HC}{7.5}$. Equating these two ratios we form the proportion

$$\frac{EH}{SO} = \frac{HC}{OJ} \text{ or with their values } \frac{8}{4} = \frac{HC}{7.5}.$$

Solve algebraically using the proportion formed.

Using the proportion

$$\frac{8}{4} = \frac{HC}{7.5}$$

Cross Multiply

$$4 \text{ HC} = (8)(7.5)$$

Simplify

$$4 \text{ HC} = 60$$

Divide both sides of the equation by 4

$$\frac{4\text{HC}}{4} = \frac{60}{4}$$

Simplify

$$\text{HC} = 15$$

Therefore, the measure of side HC is 15.

Example 4

Using the same figure in the previous page, solve for the measure of side SJ.

Solution:

First we form a ratio using a pair of corresponding sides with given values. That is $\frac{EH}{SO}$ or $\frac{8}{4}$. Next, we form a ratio using the unknown side and its corresponding side.

Since EC corresponds with SJ we form $\frac{EC}{SJ}$ or $\frac{17}{SJ}$. Now we have the ratios, equate them to form a proportion. $\frac{8}{4} = \frac{17}{SJ}$. Using this proportion, solve algebraically.

Using the proportion

Cross Multiply

Simplify

Divide both sides of the equation by 8

Simplify

$$\frac{8}{4} = \frac{17}{SJ}$$

$$8 SJ = (4)(17)$$

$$8 SJ = 68$$

$$\frac{8SJ}{8} = \frac{68}{8}$$

$$SJ = 8.5$$

Therefore, the measure of side SJ is 8.5.

Using Examples 3 and 4 we can simplify the steps in solving for the unknown sides of similar triangles.

Steps in Finding the Length of the Unknown Side(s) of Similar Triangles

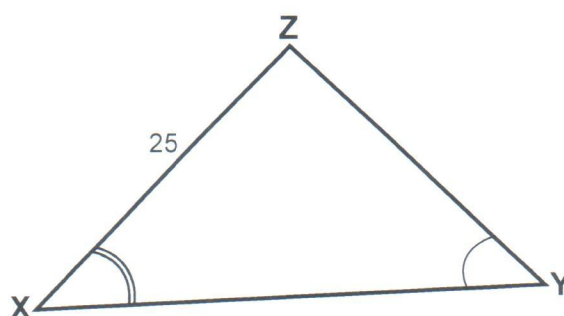
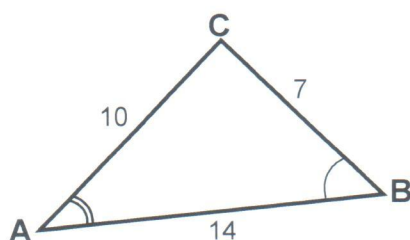
1. Form a ratio using a pair of corresponding sides with both values given.
2. Form a ratio using the unknown side and its corresponding side.
3. Using the ratios formed in (1) and (2), make a proportion.
4. Using the proportion formed in (3), Solve algebraically.

Using the simplified steps above solve the following problems.

Example 5

Given $\triangle ABC \sim \triangle XYZ$, find

- a) ZY
- b) XY



Solution:

a) Following the steps:

1. Form a ratio using a pair of corresponding sides with both values given.

$$\frac{AC}{XZ} \text{ or } \frac{10}{25}$$

2. Form a ratio using the unknown side and its corresponding side.

$$\frac{CB}{ZY} \text{ or } \frac{7}{ZY}$$

3. Using the ratios formed in (1) and (2), make a proportion.

$$\frac{10}{25} = \frac{7}{ZY}$$

4. Using the proportion formed in (3), Solve algebraically.

Using the proportion

Cross Multiply

$$10 ZY = (25)(7)$$

Simplify

$$10 ZY = 175$$

Divide both sides of the equation by 10

$$\frac{10 ZY}{10} = \frac{175}{10}$$

Simplify

$$ZY = 17.5$$

Therefore, the measure of side ZY is 17.5.

- c) Following the steps

1. Form a ratio using a pair of corresponding sides with both values given.

$$\frac{AC}{XZ} \text{ or } \frac{10}{25}$$

2. Form a ratio using the unknown side and its corresponding side.

$$\frac{AB}{XY} \text{ or } \frac{14}{XY}$$

3. Using the ratios formed in (1) and (2), make a proportion.

$$\frac{10}{25} = \frac{14}{XY}$$

4. Using the proportion formed in (3), Solve algebraically.

Using the proportion

Cross Multiply

$$10 XY = (25)(14)$$

Simplify

$$10 XY = 350$$

Divide both sides of the equation by 10

$$\frac{10 XY}{10} = \frac{350}{10}$$

Simplify

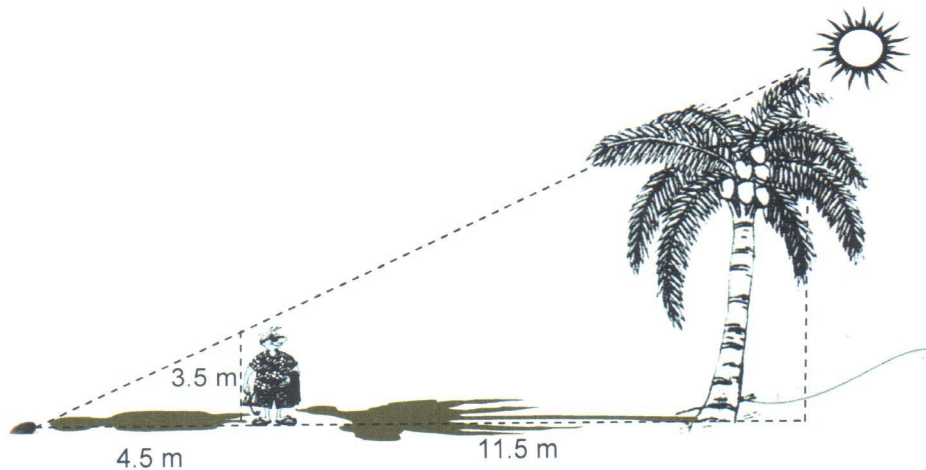
$$XY = 35$$

Therefore, the measure of side XY is 35.

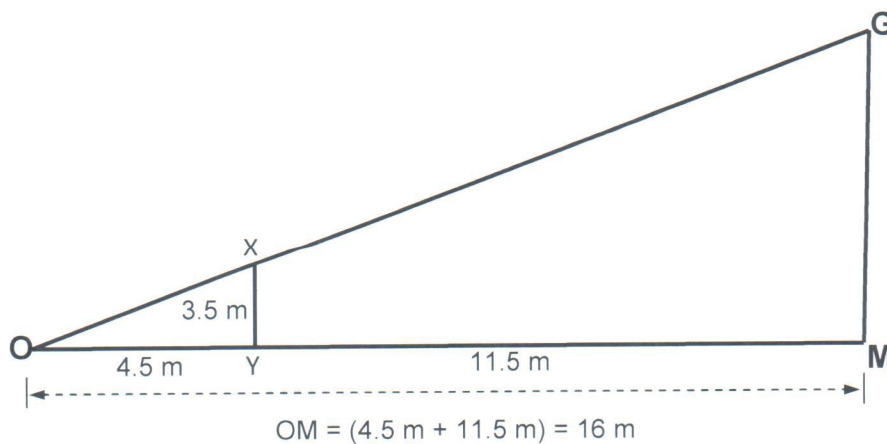
You can use knowledge on similar triangles on variety of things. One can get the actual height of a building or structure by using the triangles formed from shadows and heights of certain things. The following are examples of real-life problems which can be solved using similar triangles.

Example 6

A man 3.5 m tall stands 11.5 m away from a coconut tree. If the tip of the shadow of the man is 4.5 m. away from a stone, how tall is the tree?



From the figure above we can form overlapping triangles as drawn below. Each point in the figure can be labelled so we could name the two similar triangles.



After labelling the figure, we can now identify the two similar triangles. The bigger triangle is ΔOMG while the smaller triangle is ΔOYX .

What makes these triangles similar?

Notice that both triangles contain $\angle O$. So, we have the first pair of corresponding angles, that is, $\angle GOM \cong \angle XOY$. Another pair of corresponding angles is the right angles $\angle OYX$ and $\angle OMG$. What makes this pair of angles right? The man and the coconut tree are both standing on the same ground. Therefore it is given that their heights are perpendicular to the ground. With that we can say that $\angle OMY \cong \angle OXY$.

Now we are certain that these two triangles are similar because of the AA Similarity Theorem.

Now we are ready to answer the problem. Remember the rules? It is just as easy as 1, 2 and 3 right?

Solution:

1. Form a ratio using a pair of corresponding sides with both values given.

$$\frac{OY}{OM} \text{ or } \frac{4.5}{16}$$

2. Form a ratio using the unknown side and its corresponding side.

$$\frac{XY}{GM} \text{ or } \frac{3.5}{GM}$$

3. Using the ratios formed in (1) and (2), make a proportion.

$$\frac{4.5}{16} = \frac{3.5}{GM}$$

4. Using the proportion formed in (3), Solve algebraically.

Using the proportion

$$\frac{4.5}{16} = \frac{3.5}{GM}$$

Cross Multiply

$$4.5 GM = (16)(3.5)$$

Simplify

$$4.5 GM = 56$$

Divide both sides of the equation by 4.5

$$\frac{\cancel{4.5} GM}{\cancel{4.5}} = \frac{56}{4.5}$$

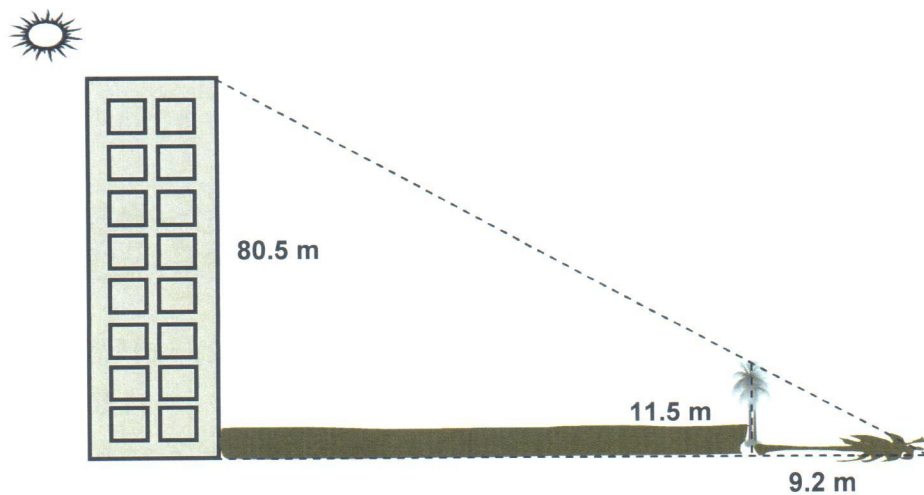
Simplify

$$GM = 12.44$$

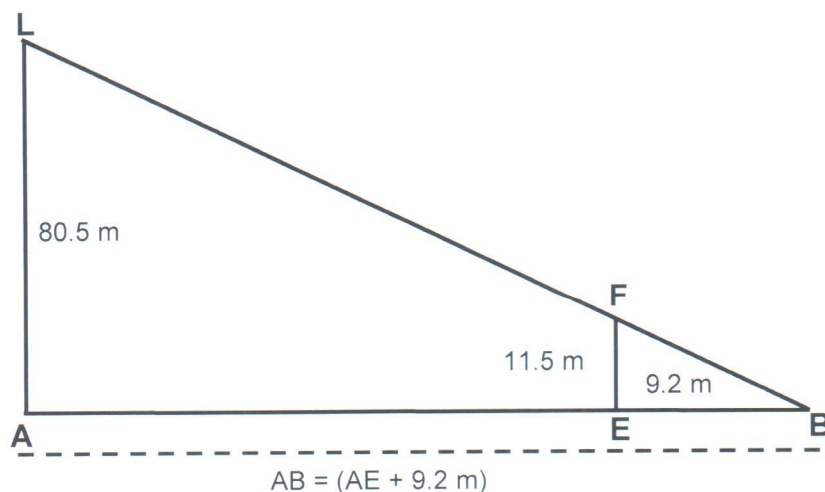
Therefore, the measure of side GM or the height of the tree is 12.44 m.

Example 7

A coconut tree 11.5 m tall creates a shadow 9.2 m long on the ground. The tree stands at the tip of the shadow of a building standing 80.5 m. How far is the tree from the building?



The same analysis in Example 6 applies in this problem. And we end up with overlapping triangles whose heights are perpendicular to the ground. Labelling the triangles we form the following figure: (Note: any letter can be used to name the points in the figure)



After labelling the figure, we can now identify the two triangles. The bigger triangle is $\triangle LAB$ and the smaller triangle is $\triangle FEB$. Since $\angle LAB \cong \angle FEB$ and $\angle LBA \cong \angle FBE$, we therefore conclude that $\triangle LAB \sim \triangle FEB$ through AA Similarity Theorem.

Solution:

1. Form a ratio using a pair of corresponding sides with both values given.

$$\frac{LA}{FE} \text{ or } \frac{80.5}{11.5}$$

2. Form a ratio using the unknown side and its corresponding side.

$$\frac{AB}{EB} \text{ or } \frac{(AE + 9.2)}{9.2}$$

3. Using the ratios formed in (1) and (2), make a proportion.

$$\frac{80.5}{11.5} = \frac{(AE + 9.2)}{9.2}$$

4. Using the proportion formed in (3), solve algebraically.

Using the proportion

$$\frac{80.5}{11.5} = \frac{(AE + 9.2)}{9.2}$$

Cross Multiply

$$(80.5)(9.2) = (11.5)(AE + 9.2)$$

Simplify (Use distributive property)

$$740.60 = 11.5 AE + 105.80$$

Transpose 11.5 to the left side

$$740.60 - 105.80 = 11.5 AE$$

$$634.80 = 11.5 AE$$

Divide both sides of the equation by 11.

$$\frac{634.80}{11.5} = \frac{11.5 AE}{11.5}$$

Simplify

$$55.20 = AE$$

Therefore, the measure of side AE which is the distance between the building and the coconut tree is 55.20 m.

Can you think of other real-life problems which can be solved using similar triangles?

Yes you are right if you are thinking that there are so many real life situations where the knowledge in this lesson can be used. Are you now ready to solve the next practice exercise?

Great!

Now, review the lesson and be ready to solve the next Practice Exercise.

NOW DO PRACTICE EXERCISE 4

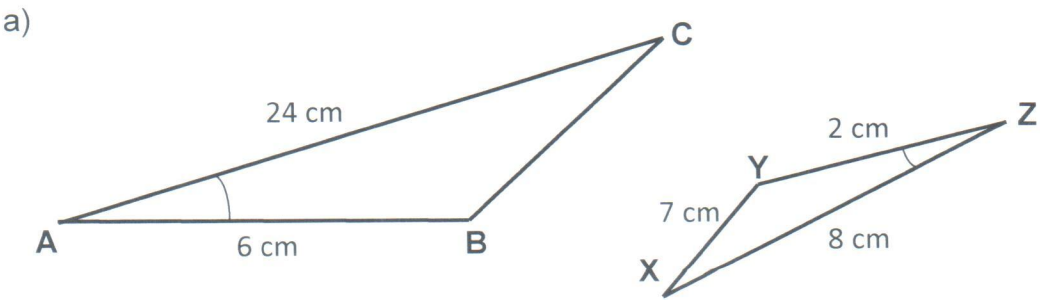


Practice Exercise 4

- 1) Given $\triangle LAB \sim \triangle YOU$, Identify the pairs of corresponding angles and pairs of corresponding sides in the following figures.

Given	Pairs of Corresponding Angles	Pairs of Corresponding Sides
a) 		
b) 		

- 2) Identify the theorem that justifies the similarity of the following triangles on the next page. Write the answer on the line provided. After Identifying the Similarity Theorem used, solve for the missing measures (length) of sides of the triangles.



Similarity Theorem used: _____

Solution:

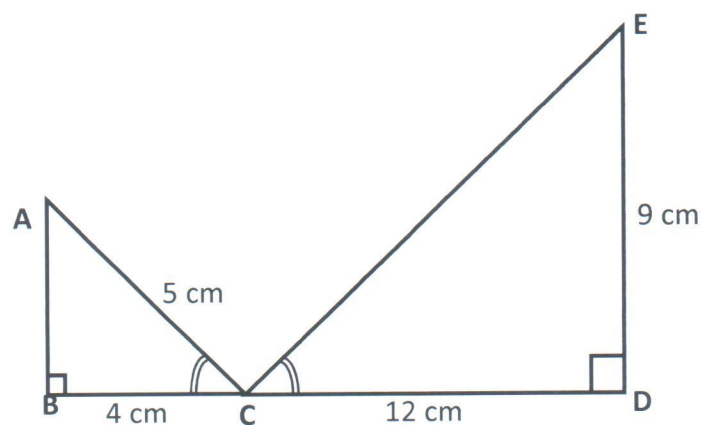
Step 1:

Step 2:

Step 3:

Step 4:

b)



Similarity Theorem used: _____

Solution:

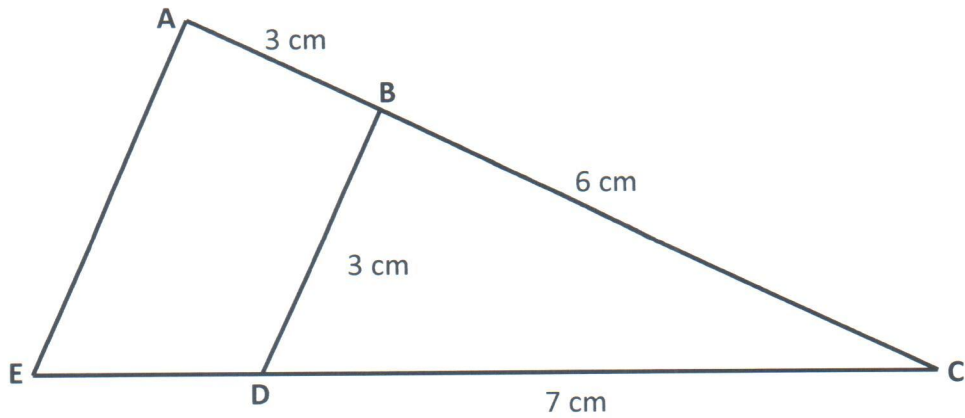
Step 1:

Step 2:

Step 3:

Step 4:

c)



Similarity Theorem used: _____

Solution:

Step 1:

Step 2:

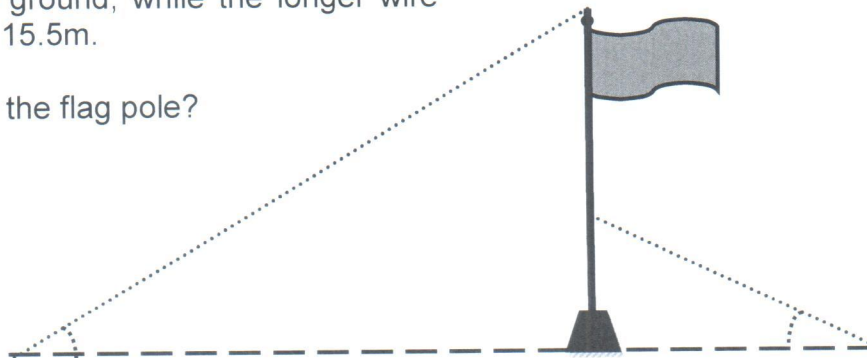
Step 3:

Step 4:

- 3) Solve the following real-life related problem. Show the sketch and label it properly. (You can use any letters to name the points)

A flag pole is being suspended by wires on both sides for support. The shorter wire measures 6.5 m and it is tied up 4 m above the ground, while the longer wire measures 15.5 m.

How tall is the flag pole?



Draw your sketch here and label it properly.

Solution

Step 1:

Step 2:

Step 3:

Step 4:

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

TOPIC 1: SUMMARY

This summarizes some of the important ideas and concepts to remember.

- Two lines that meet at a common point called **point of intersection** are known as **Intersecting Lines**.
- Two angles are said to be **adjacent** if they share a common side and a common vertex.
- Two angles are said to be **complementary** if the sum of their measures is **exactly 90°** .
- Two angles are said to be **supplementary** if the sum of their measures is **exactly 180°** .
- A **linear pair** refers to two angles formed by two intersecting lines which are always supplementary and adjacent with each other.
- Two angles formed by two intersecting lines which are opposites and share a common vertex are called **vertical angles**.
- A line which intersects two or more parallel lines is known as a **transversal line**.
- **Interior angles** (co-interior angles) are formed when parallel lines are cut by a transversal.
- **Alternate angles** are formed when parallel lines are cut by a transversal. These angles lie on opposite sides of the transversal line. Both must either be in the interior (Alternate Interior Angles) or on the exterior (Alternate Exterior Angles) sides of the parallel lines.
- **Alternate Interior Angles Theorem:** If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.
- **Alternate Exterior Angles Theorem:** If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.
- **Corresponding angles** are formed when parallel lines are cut by a transversal. Specifically these angles lie on the same side of the transversal however, one is an interior and the other is an exterior angle.
- **Corresponding angles postulate:** If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.
- **Triangle Angle Sum Theorem:** The sum of the measures of the interior angles of a triangle is 180° .
- **Exterior Angle Theorem:** The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.
- Two triangles are similar if their corresponding or matching angles are congruent or they have equal measures and corresponding sides are in proportion.
- The following are the theorems which proves triangle similarities: AA Similarity Theorem, SSS (Side-Side-Side) Similarity Theorem and (Side-Angle-Side) Similarity Theorem:

REVISE LESSON 1 – 4. THEN DO TOPIC TEST 1 IN ASSIGNMENT BOOK 5.

ANSWERS PRACTICE EXERCISES 1-4

Practice Exercise 1

1. a) True f) False
 b) False g) True
 c) True h) True
 d) False i) False
 e) False j) True

 2. a) Yes.
 b) No. The lines are parallel with each other.
 c) No. The line intersects a curve.

 3. a) ℓ_3 (other possible names: line 3, $\overleftrightarrow{FD}$, $\overleftrightarrow{FY}$)
 b) $\angle BXC$ and $\angle AXE$ or $\angle AXY$
 c) i. 125° ii 125° iii. 55°
 d) $\angle AXB$ & $\angle CXE$ and $\angle AXE$ & $\angle BXC$
 e) $\angle AXB$ & $\angle BXC$ and $\angle BXC$ & $\angle CXE$
 $\angle AXB$ & $\angle AXE$ and $\angle AXE$ & $\angle CXE$
 f) i. Vertical Angles
 ii. Linear Pair Angles
 iii. Linear Pair Angles
 iv. None
 v. Vertical Angles
-

Practice Exercise 2

1. a) ℓ_1 and ℓ_2
 b) ℓ_1
 c) ℓ_1
 d) $\angle 9$ & $\angle 11$ and $\angle 10$ & $\angle 12$
 e) $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$
 f) $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$
 g) i. Linear Pair Angles or Adjacent Angles
 ii. Alternate Interior Angles
 iii. Alternate Exterior Angles
 iv. Vertical Angles
 v. Linear Pair Angles or Adjacent Angles
 vi. Corresponding Angles
 vii. Linear Pair Angles or Adjacent Angles

2. a) 55°
 b) 55°
 c) 140°
 d) 45°
 e) 150°
3. a) $\alpha = 98^\circ$ $\beta = 82^\circ$
 b) $\alpha = 90^\circ$ $\beta = 90^\circ$
 c) $\alpha = 60^\circ$ $\beta = 60^\circ$
 d) $\alpha = 60^\circ$ $\beta = 120^\circ$

Practice Exercise 3

1.

Given Triangle	Classification defined by	
	length of sides	angles contained
a)	Isosceles Triangle	Right Triangle
b)	Scalene Triangle	Obtuse Triangle
c)	Scalene Triangle	Right Triangle
d)	Isosceles Triangle	Obtuse Triangle
e)	Scalene Triangle	Acute Triangle

2. a) 105°
 b) 75°
3. a) 39°
 b) 48°

Practice Exercise 4

1.

Given	Pairs of Corresponding Angles	Pairs of Corresponding Sides
a	$\angle L$ and $\angle U$ $\angle B$ and $\angle Y$ $\angle A$ and $\angle O$	$\overline{LA}$ and $\overline{UO}$ $\overline{LB}$ and $\overline{UY}$ $\overline{AB}$ and $\overline{OY}$
b	$\angle L$ and $\angle Y$ $\angle B$ and $\angle O$ $\angle A$ and $\angle U$	$\overline{LA}$ and $\overline{YU}$ $\overline{LB}$ and $\overline{YO}$ $\overline{AB}$ and $\overline{UO}$

2) a) Similarity Theorem used: SAS

Solution:

Step 1: Form a ratio using a pair of corresponding sides with both values given.

$$\frac{AC}{XZ} = \frac{24}{8}$$

Step 2: Form a ratio using the unknown side and its corresponding side

$$\frac{BC}{XY} = \frac{BC}{7}$$

Step 3: Using the ratios formed in (1) and (2), make a proportion.

$$\frac{BC}{7} = \frac{24}{8}$$

Step 4: Using the proportion formed in (3), Solve algebraically

$$\begin{aligned} BC &= \frac{24(7)}{8} \\ &= 21 \text{ cm} \end{aligned}$$

b) Similarity Theorem used: ASA

Solution:

Step 1: Form a ratio using a pair of corresponding sides with both values given.

$$\frac{BC}{CD} = \frac{4}{12}$$

Step 2: Form a ratio using the unknown side and its corresponding side

$$\frac{AC}{CE} = \frac{5}{CE}$$

Step 3: Using the ratios formed in (1) and (2), make a proportion.

$$\frac{4}{12} = \frac{5}{CE}$$

Step 4: Using the proportion formed in (3), Solve algebraically

$$\begin{aligned} CE &= \frac{12(5)}{4} \\ &= 15 \text{ cm} \end{aligned}$$

d) Similarity Theorem used: SAS

Solution:

Step 1: Form a ratio using a pair of corresponding sides with both values given.

$$\frac{AC}{BC} = \frac{9}{6}$$

Step 2: Form a ratio using the unknown side and its corresponding side

$$\frac{AE}{BD} = \frac{AE}{3}$$

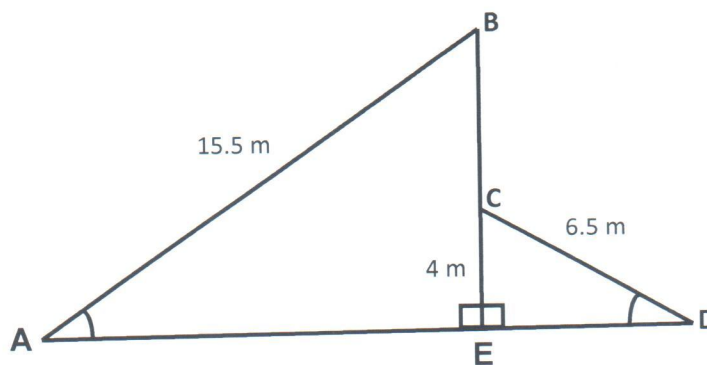
Step 3: Using the ratios formed in (1) and (2), make a proportion.

$$\frac{AE}{3} = \frac{9}{6}$$

Step 4: Using the proportion formed in (3), Solve algebraically

$$\begin{aligned} AE &= \frac{3(9)}{6} \\ &= 4.5 \text{ cm} \end{aligned}$$

3)



Solution:

Step 1: Form a ratio using a pair of corresponding sides with both values given.

$$\frac{AB}{CD} = \frac{15.5}{6.5}$$

Step 2: Form a ratio using the unknown side and its corresponding side

$$\frac{BE}{CE} = \frac{BE}{4}$$

Step 3: Using the ratios formed in (1) and (2), make a proportion.

$$\frac{15.5}{6.5} = \frac{BE}{4}$$

Step 4: Using the proportion formed in (3), Solve algebraically

$$\begin{aligned} BE &= \frac{4(15.5)}{6.5} \\ &= 9.5 \text{ m} \end{aligned}$$

END OF TOPIC 1

TOPIC 2

PYTHAGORAS THEOREM

Lesson 5:	Square and Square Roots
Lesson 6:	Pythagoras' Theorem
Lesson 7:	Finding the Hypotenuse
Lesson 8:	Finding the Adjacent and Opposite Sides
Lesson 9:	Solving Problems Involving Pythagoras' Theorem

TOPIC 2: PHYTAGORAS' THEOREM

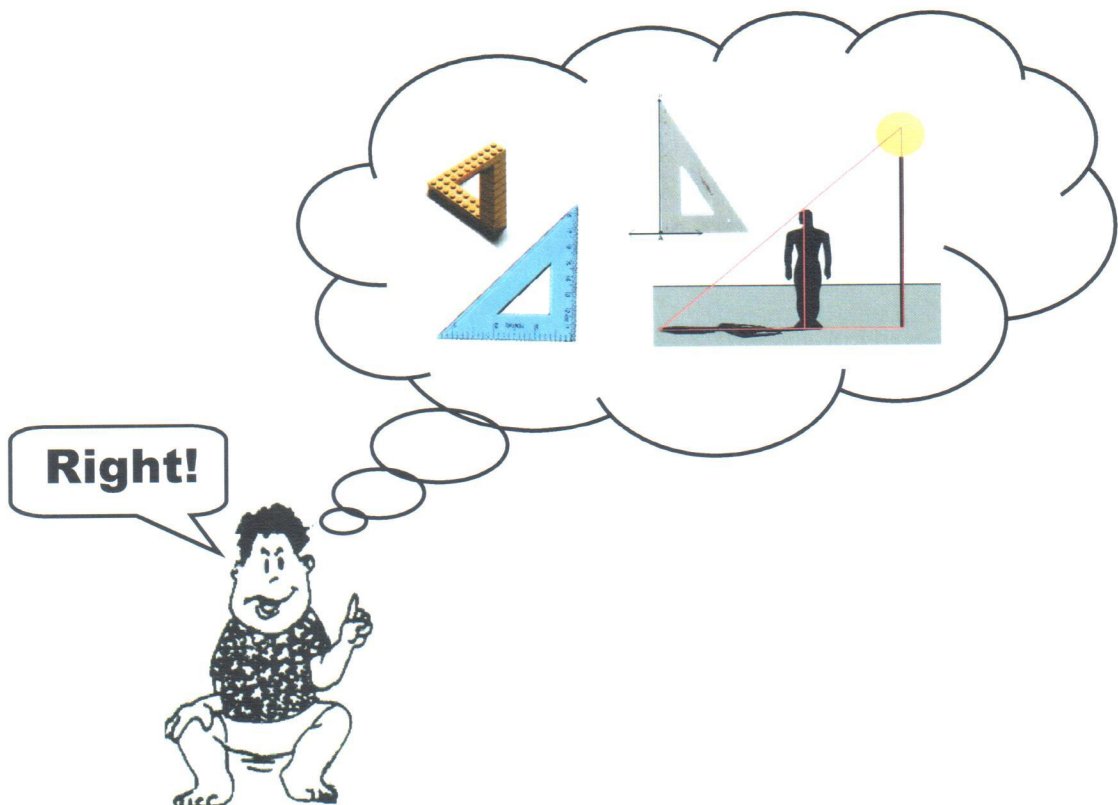
Introduction



When you hear of right angled triangles, what comes up your mind first?

Did you remember the blocks of Lego you built when you were a child?

Or perhaps it reminds you of the triangular ruler your mom bought you in your elementary days.



For some, this reminds them of one of the greatest Mathematicians in history, Pythagoras!

In this topic you will learn more than what a right triangle is. You will be lead to its useful and practical applications using the Pythagoras' Theorem. You will see how knowledge on right triangle will help you navigate and accurately measure objects, distances and heights. This fundamental knowledge will also prepare you for your higher Mathematics such as Trigonometry, Analytic Geometry and Calculus.

Lesson 5: Square and Square Roots



You learnt something about triangles in the previous topic which will be very helpful in learning this new topic.



In this lesson you will:

- define “square of a number”
- define “square roots”
- find the square and square roots using tables

The Square of a Number

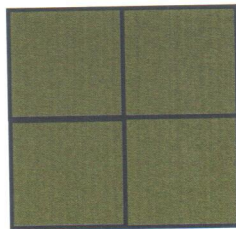
In basic Geometry, you have learnt that a square is a quadrilateral with four equal sides. Is there anything else you think of about a square? Do you visualize a square as this?



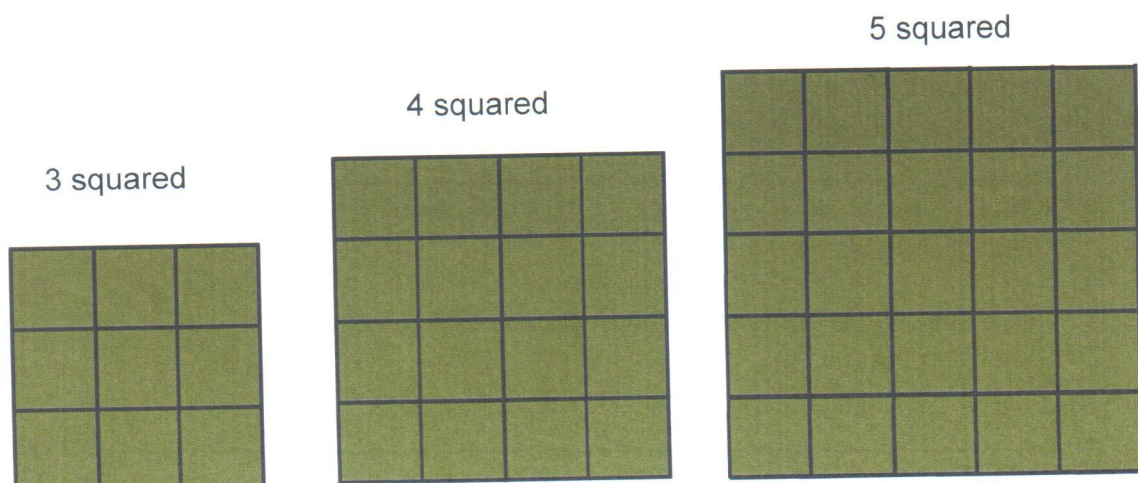
Then you are right. Have you heard of squaring a number?

If you are told to square the number 2, how does it look like?

You are right if you think this way. A square with each side measures 2 units.



Therefore, we think of the succeeding numbers this way



If we will organize our observations in the previous page, we will have this table:

Number (length of side per unit)	Total number of squares
1	1
2	4
3	9
4	16
5	25

How did we get the values in the second column on the table above?
You are right, if you say through counting.

What if you are asked to get the square of 10? Square of 15? Would it be too time consuming to draw?

Observe the set of numbers in the second column in the table above. How are they related with the numbers on the first column?

Yes you are right if you think that the numbers on the second column are the products when the first number (found on the first column of the table) is multiplied by itself.

Let us verify.

$$\begin{aligned}1 \times 1 &= 1 \\2 \times 2 &= 4 \\3 \times 3 &= 9 \\4 \times 4 &= 16 \\5 \times 5 &= 25\end{aligned}$$

So, if you are now asked to get the square of 10, you will just multiply 10 by itself. That is, $10 \times 10 = 100$. It's easy right?

With that we can now say that

The square of a number is the product when the number is multiplied by itself.

In symbols we can say that the square of a number n is

$$n \times n \text{ or } n^2$$

Example 1

Complete the table below by supplying the square of the given number

Number	Square of a Number
6	
7	
8	
9	
10	

Solution:

36, 49, 64, 81, 100

Example 2

Find the square of the following:

- a) 11
- b) 16
- c) 23
- d) 25
- e) 1.2
- f) 1.5
- g) $\frac{1}{2}$
- h) $\frac{2}{3}$

Solution:

- a) $11 = 11^2$ or $11 \times 11 = \mathbf{121}$
- b) $16 = 16^2$ or $16 \times 16 = \mathbf{256}$
- c) $23 = 23^2$ or $23 \times 23 = \mathbf{529}$
- d) $25 = 25^2$ or $25 \times 25 = \mathbf{625}$

Numbers which are not whole such as decimals and fractions can also be squared as long as we follow the formula, that is by raising a number to the power of 2 or just multiply the number by itself.

a) $1.2 = (1.2)^2$ or $1.2 \times 1.2 = \mathbf{1.44}$

b) $1.5 = (1.5)^2$ or $1.5 \times 1.5 = \mathbf{2.25}$

c) $\frac{1}{2} = \left(\frac{1}{2}\right)^2$ or $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

d) $\frac{2}{3} = \left(\frac{2}{3}\right)^2$ or $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

When a whole number is raised to the second power or it is squared, the result is a **perfect square number**.

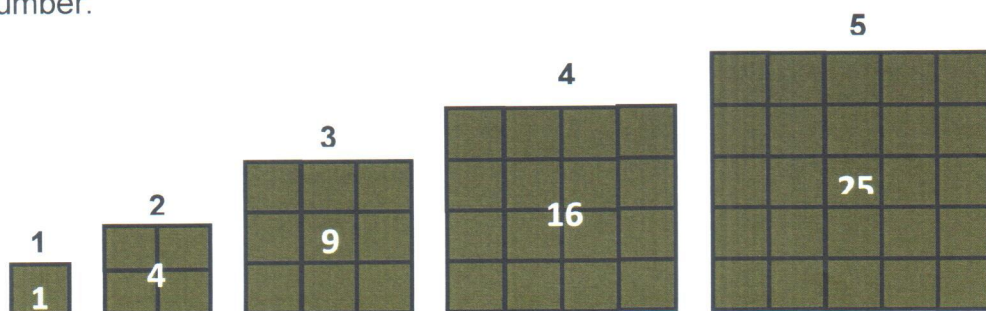
Square Root of a Number

After you have mastered the ways of getting the square of a number, you are now ready to find the square root of a number.

But what does “square root” exactly mean?

The **square root** of a number is a number that, when multiplied by itself, gives the number. In symbols, we can write the square root of a number n as $\sqrt{n}$.

To visualize the process, let us use the figures we used earlier in getting the square of a number.



If we are asked to get the square root of 1, the answer is 1. How about the square root of 4? Look at the second box above, there are 4 square units in the bigger square, each side of the square is comprised of 2 square units. Therefore, we say that the square root of 4 is 2. In addition, notice that if 2 is multiplied by itself, it gives the given number which is 4. How about getting the following: $\sqrt{9}$, $\sqrt{16}$, and $\sqrt{25}$.

Summarizing the observations in a table, we get

Number	Square Root
1	1
4	2
9	3
16	4
25	5

What did you notice in the table?

Excellent if you find the values are related. The square of 2 is 4, while the square root of 4 is 2. The idea is also true for all values in the table.

Example 3

Complete the table below by supplying the square root of the given number

Number	Square Root
36	
49	
64	
81	
100	

Solution:

6, 7, 8, 9, 10

Example 4

Find the square roots of the following numbers:

a) $\sqrt{121}$

c) $\sqrt{0.81}$

b) $\sqrt{144}$

d) $\sqrt{\frac{4}{9}}$

Solution:

a) $\sqrt{121} = 11$

b) $\sqrt{144} = 12$

c) $\sqrt{0.81} = 0.9$ Getting the square root of decimal numbers is just like getting the roots of whole numbers. **The number of decimal places must be considered.**

d) $\sqrt{\frac{4}{9}} = \frac{2}{3}$ Getting the square roots of fractions is also simple. Just get the square root of the numerator over the square root of the denominator.

The square root of perfect square number is a whole number. For some numbers which are not perfect square, refer to the table on the next page .

Table of Squares and Square Roots of Numbers 1-150

No.	Square	Square Root	No.	Square	Square Root	No.	Square	Square Root
1	1	1.000	51	2,601	7.141	101	10,201	10.050
2	4	1.414	52	2,704	7.211	102	10,404	10.100
3	9	1.732	53	2,809	7.280	103	10,609	10.149
4	16	2.000	54	2,916	7.348	104	10,816	10.198
5	25	2.236	55	3,025	7.416	105	11,025	10.247
6	36	2.449	56	3,136	7.483	106	11,236	10.296
7	49	2.646	57	3,249	7.550	107	11,449	10.344
8	64	2.828	58	3,364	7.616	108	11,664	10.392
9	81	3.000	59	3,481	7.681	109	11,881	10.440
10	100	3.162	60	3,600	7.746	110	12,100	10.488
11	121	3.317	61	3,721	7.810	111	12,321	10.536
12	144	3.464	62	3,844	7.874	112	12,544	10.583
13	169	3.606	63	3,969	7.937	113	12,769	10.630
14	196	3.742	64	4,096	8.000	114	12,996	10.677
15	225	3.873	65	4,225	8.062	115	13,225	10.724
16	256	4.000	66	4,356	8.124	116	13,456	10.770
17	289	4.123	67	4,489	8.185	117	13,689	10.817
18	324	4.243	68	4,624	8.246	118	13,924	10.863
19	361	4.359	69	4,761	8.307	119	14,161	10.909
20	400	4.472	70	4,900	8.367	120	14,400	10.954
21	441	4.583	71	5,041	8.426	121	14,641	11.000
22	484	4.690	72	5,184	8.485	122	14,884	11.045
23	529	4.796	73	5,329	8.544	123	15,129	11.091
24	576	4.899	74	5,476	8.602	124	15,376	11.136
25	625	5.000	75	5,625	8.660	125	15,625	11.180
26	676	5.099	76	5,776	8.718	126	15,876	11.225
27	729	5.196	77	5,929	8.775	127	16,129	11.269
28	784	5.292	78	6,084	8.832	128	16,384	11.314
29	841	5.385	79	6,241	8.888	129	16,641	11.358
30	900	5.477	80	6,400	8.944	130	16,900	11.402
31	961	5.568	81	6,561	9.000	131	17,161	11.446
32	1,024	5.657	82	6,724	9.055	132	17,424	11.489
33	1,089	5.745	83	6,889	9.110	133	17,689	11.533
34	1,156	5.831	84	7,056	9.165	134	17,956	11.576
35	1,225	5.916	85	7,225	9.220	135	18,225	11.619
36	1,296	6.000	86	7,396	9.274	136	18,496	11.662
37	1,369	6.083	87	7,569	9.327	137	18,769	11.705
38	1,444	6.164	88	7,744	9.381	138	19,044	11.747
39	1,521	6.245	89	7,921	9.434	139	19,321	11.790
40	1,600	6.325	90	8,100	9.487	140	19,600	11.832
41	1,681	6.403	91	8,281	9.539	141	19,881	11.874
42	1,764	6.481	92	8,464	9.592	142	20,164	11.916
43	1,849	6.557	93	8,649	9.644	143	20,449	11.958
44	1,936	6.633	94	8,836	9.695	144	20,736	12.000
45	2,025	6.708	95	9,025	9.747	145	21,025	12.042
46	2,116	6.782	96	9,216	9.798	146	21,316	12.083
47	2,209	6.856	97	9,409	9.849	147	21,609	12.124
48	2,304	6.928	98	9,604	9.899	148	21,904	12.166
49	2,401	7.000	99	9,801	9.950	149	22,201	12.207
50	2,500	7.071	100	10,000	10.000	150	22,500	12.247

Note that **the square of a whole number is always a whole number. The square root of a whole number is not always a whole number.** Only the perfect square numbers result to a whole number square root. The highlighted numbers are perfect square numbers that yield to a whole number root.

Example 5

Using the table of Squares and Square Roots of numbers 1-150, give the square roots of the following: (round off the answer to the nearest tenths)

a) $\sqrt{7} = 2.646 = \mathbf{2.6}$

b) $\sqrt{32} = 5.657 = \mathbf{5.7}$

c) $\sqrt{88} = 9.381 = \mathbf{9.4}$

d) $\sqrt{135} = 11.619 = \mathbf{11.6}$

e) $\sqrt{147} = 12.124 = \mathbf{12.1}$

NOW DO PRACTICE EXERCISE 5

**Practice Exercise 5**

1) Find the square of the following: (You may use the table for squares)

a) 9 _____

b) 12 _____

c) 0.9 _____

d) 25 _____

e) 1.2 _____

2) Find the square root of the following (round off answers to the nearest tenth)

a) $\sqrt{34}$ _____

b) $\sqrt{121}$ _____

c) $\sqrt{85}$ _____

d) $\sqrt{134}$ _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.
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Lesson 6: Pythagoras' Theorem



You learnt something about right triangles in Strand 2 of Grade 8 Mathematics.



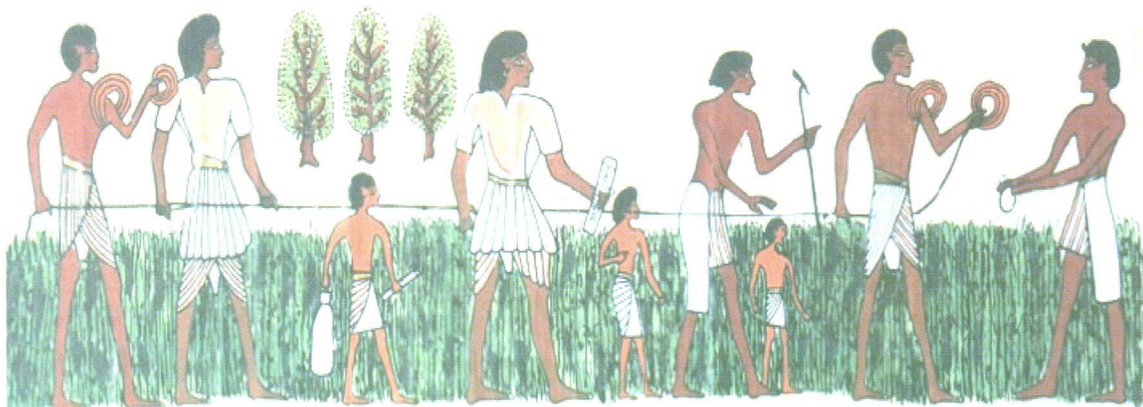
In this lesson, you will:

- describe a right-angled triangle and its parts
- state the Pythagoras' Theorem
- explain the relationship between Pythagoras' theorem and length of the sides of a right-angled triangle

History of the Pythagoras Theorem

Pythagoras was born in the late 6th century B.C. on the island of Samos of a Phoenician mother and a Greek stonecutter father. He was known as one of the truly genius and influential people in the field of Mathematics.

At a young age he left home to travel in quest of the knowledge and learn everything he could find. He studied under the Greek named Thales who was just beginning to discover the concepts of geometry. Encouraged by Thales, the young Pythagoras travelled places which include Babylon and Egypt.



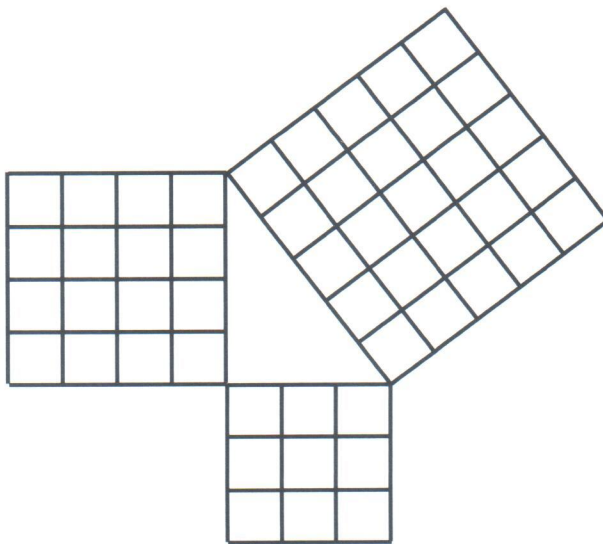
In Egypt he studied with the people known as the "rope-stretchers". These were the engineers who built the pyramids. They held a very special secret in the form of a rope tied in a circle with 12 evenly spaced knots. It turns out that if the rope was pegged to the ground in the dimensions of 3-4-5, a right triangle would emerge instantly. This enabled them to lay the foundations for their buildings accurately.

He travelled to all the known parts of the Mediterranean world. During his travels he came to the conclusion that the earth must be round. In history, he is given credit as the first person to spread this idea.

He spent many years learning by travelling. He returned home to share what he knew, but the people of his home town Samos were less than enthusiastic.

He became tired of finding no one who would listen to him. He decided to "buy" a student. He found a homeless child and offered him a bribe. Pythagoras would pay him for every lesson the boy mastered. The boy thought this was great because he could make better wages than in a whole day's work in the hot sun by just listening to the old man. The boy concentrated hard while Pythagoras introduced him to mathematical disciplines.

Pythagoras led the boy from the simple calculations of the Egyptian rope-stretchers, to the methods of the Phoenician navigators, to abstract rules and reasoning. Soon the subject became so interesting that the boy begged for more and more lessons. At this point, Pythagoras explained that he could not afford to pay someone to just listen to him anymore. So they reached a bargain. The boy had saved enough to pay Pythagoras for his lessons. Probably this started Pythagoras' organized education. Eventually Pythagoras left the island of Samos and settled on the Isle of Croton. This is where he formed his Secret Brotherhood. It was a religious order with initiation rites and purifications and Pythagoras was its supreme unquestioned leader. He taught them that knowledge was the greatest purification, and that knowledge meant mathematics.



The most famous discovery that Pythagoras made came from his fascination with the Egyptian 3-4-5 rope-stretchers triangle. He spent years thinking about it and what magic it might hold. One day while drawing in the sand he found that if a square is drawn from each side of the 3-4-5 triangle, the area of the two small squares added together equals the area of the large square.

When he revealed this finding to his followers, he used the general terms of ***a*** & ***b*** for the shorter legs and ***c*** for the longer side which he gave the name "hypotenuse". Thus we have the famous Pythagoras' Theorem or popularly known as the Pythagorean Theorem.

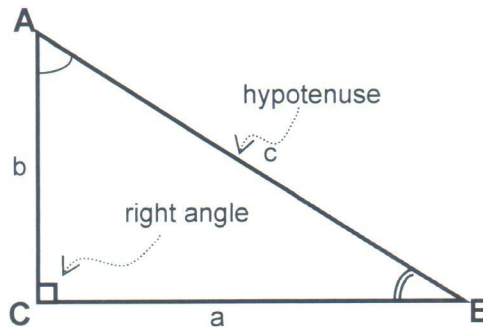
A Closer look to a right triangle

General characteristics of every triangle include having three (3) sides and three (3) angles. The sum of all three angles is 180° .

A Right Triangle contains one right angle and two acute angles.

In triangle ABC as shown in the figure below, $\angle A$ and $\angle B$ are acute angles and $\angle C$ is a right angle.

Sides **a** or $\overline{CB}$ and **b** or $\overline{AC}$ are the legs of the triangle while side **c** or $\overline{AB}$ is called as the hypotenuse.



The hypotenuse is the longest side of a right triangle and it is located opposite the right angle.

Using the right triangle drawn above the

Pythagoras Theorem is defined as “the square of the hypotenuse is equal to the sum of the squares of the two legs” denoted by:

$$c^2 = a^2 + b^2$$

It means that we get the square of the hypotenuse and the squares of the two legs. If you added the squares of the two legs it must be equal to the square of the hypotenuse, then the triangle is a right angled triangle.

Example 1

Let us prove that triangle ABC is a right-angled triangle using the Pythagoras Theorem and the following information:

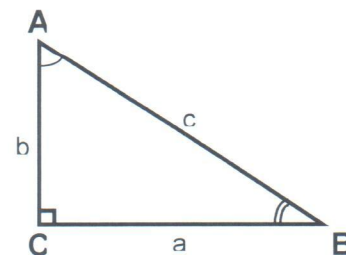
In triangle ABC, $a = 4$, $b = 3$ and $c = 5$.

Since side c is the hypotenuse we use

Substitute the values

Get the square of the hypotenuse and sides of the triangle

Simplify



$$c^2 = a^2 + b^2$$

$$5^2 = 4^2 + 3^2$$

$$25 = 16 + 9$$

$$25 = 25$$

Since $25 = 25$, it means that both sides of the equation are equal, **we conclude that ABC is a right-angled triangle.**

Example 2

Given the triangle ABC, with $a = 4$ cm, $b = 6$ cm and $c = 8$ cm, test if the figure will form a right-angled triangle.

Since c is the longest side with a measure of 8 cm, we consider it as the hypotenuse, thus we test the triangle using

$$\begin{aligned}(8\text{cm})^2 &\stackrel{?}{=} (4\text{cm})^2 + (6\text{cm})^2 \\ 64\text{ cm}^2 &\stackrel{?}{=} 16\text{ cm}^2 + 36\text{ cm}^2 \\ 64\text{ cm}^2 &\neq 52\text{ cm}^2\end{aligned}$$

64 cm^2 is not equal to 52 cm^2 . We therefore conclude that **the triangle formed by the sides a, b and c is not a right angled triangle.**

Example 3

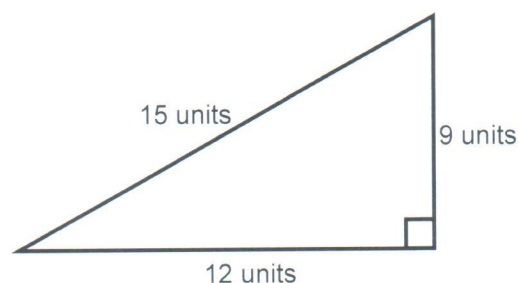
Given a triangle whose sides' measure 9, 12 and 15 units respectively, test if it is a right-angled triangle. Then, draw and label the triangle accordingly.

Since 15 units is the measure of the longest side, we assume that it is the possible hypotenuse of the triangle and 9 & 12 units respectively are the measures of the two legs.

Using the Pythagoras' theorem, we substitute

$$\begin{aligned}15^2 &\stackrel{?}{=} 9^2 + 12^2 \\ 225 &\stackrel{?}{=} 81 + 144 \\ 225 &= 225\end{aligned}$$

Since both sides of the equation are equal, **we therefore, conclude that the given measures belong to a right angled triangle** drawn and labelled as follows:



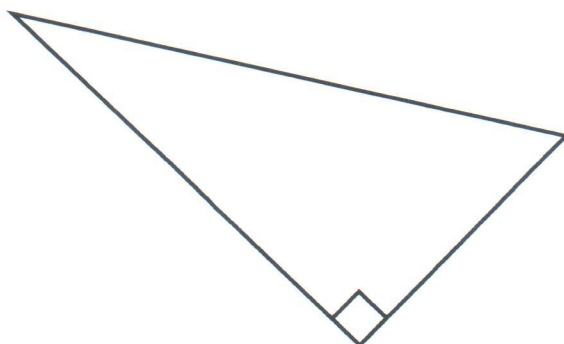
Since 15 units is the longest measure of the sides, it is therefore considered as the hypotenuse. 9 and 12 units represent the measures of the shorter and longer legs respectively.

NOW DO PRACTICE EXERCISE 6

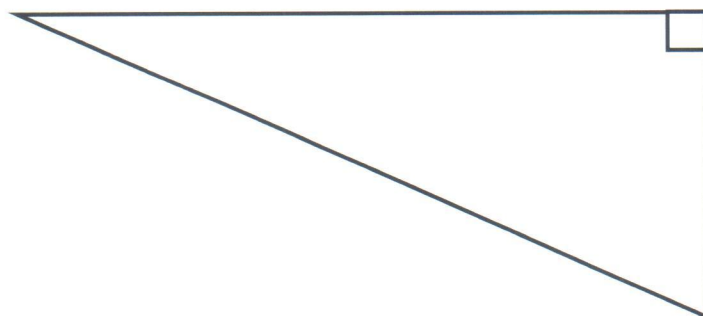
**Practice Exercise 6**

1. Label the following right-angled triangles using the given information:

a) $a = 7 \text{ cm}$ $b = 24 \text{ cm}$ $c = 25 \text{ cm}$



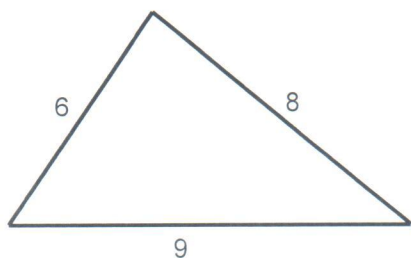
b) $x = 6 \text{ m}$ $y = 8 \text{ m}$ $z = 10 \text{ m}$



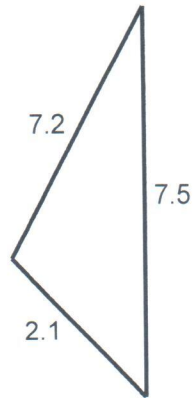
2. Do the following lengths form a right triangle? Support your answer with a proof.

a)

Answer: _____
Proof: _____



b)



Answer: _____

Proof: _____

-
3. Draw a right triangle. Name the right angle as X. Now name the other points as A and B. Then name the following:

Hypotenuse: _____

Legs(sides) : _____ and _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 7: Finding the Hypotenuse



You learnt about what a right triangle is and the Pythagorean Theorem.

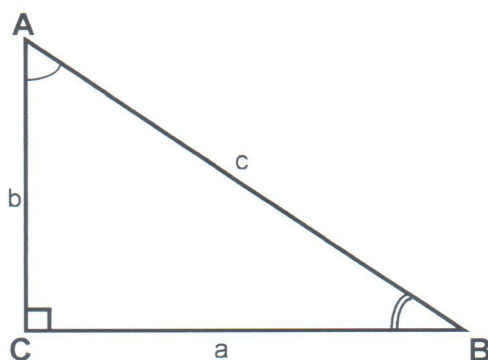


In this lesson, you will:

- apply the Pythagorean Theorem in finding the measure of the hypotenuse.

You have learnt in the previous lesson the Pythagorean Theorem.

To solve for the hypotenuse, we derive the formula by using the Pythagoras Theorem defined by the right triangle ABC as drawn below:



We use the formula represented as

$$c^2 = a^2 + b^2$$

Get the square roots of both sides of the equation

$$\sqrt{c^2} = \sqrt{a^2 + b^2}$$

Simplify

$$c = \sqrt{a^2 + b^2}$$

Therefore, we already derived the formula in solving for the hypotenuse.

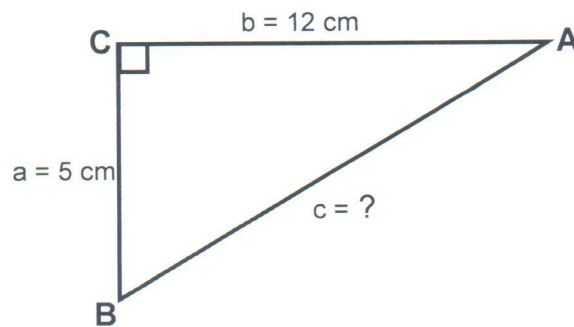
To solve for the measure of the hypotenuse given a right triangle ACB wherein C is the right angle, we get **the square root of the sum of the squares of the measures of the two legs**. In symbol we represent the formula as:

$$c = \sqrt{a^2 + b^2}$$

Wherein **c** is the measure of the hypotenuse and **a** and **b** are the measures of the legs respectively.

Example 1

Using the formula in solving for the hypotenuse of a right triangle, solve for the measure of the hypotenuse of triangle ACB



Solution:

Since the unknown side is the hypotenuse, we use

$$c^2 = 5^2 + 12^2 \quad (\text{you may use the table of squares and square roots on page 58})$$

$$c = \sqrt{25+144}$$

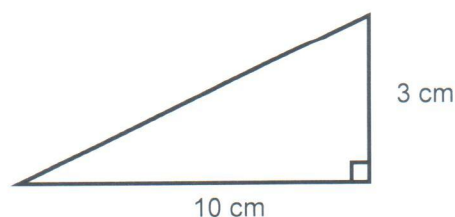
$$c = \sqrt{169} \quad (\text{you may use the table of squares and square roots on page 58})$$

$$c = 13 \text{ cm}$$

The measure of the hypotenuse of $\triangle ACB$ is 13 cm.

Example 2

Find the measure of the hypotenuse in the figure



Solution:

Since the triangle is not labelled, we represent the unknown measure of hypotenuse as x . Then we directly substitute the measures of the legs in the formula.

$$x = \sqrt{3^2 + 10^2} \quad (\text{you can also write } \sqrt{10^2 + 3^2}, \text{ anyway interchanging the position does not affect the sum because Addition is commutative.})$$

$$x = \sqrt{9+100}$$

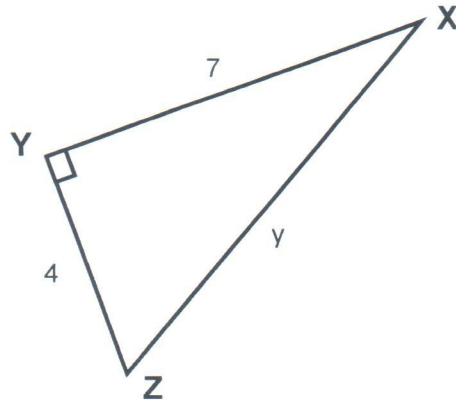
$$x = \sqrt{109} \quad (\text{you may use the table of squares and square roots on page 58})$$

$$x = 10.44 \text{ cm}$$

Therefore, the measure of the hypotenuse is 10.44 cm.

Example 3

Find the measure of the hypotenuse in triangle XYZ



Solution:

Looking at the figure, the hypotenuse is side y or $\overline{XZ}$. Following the formula which states that **the measure of the hypotenuse is equal to the square root of the sum of the squares of the measures of the two legs**

We write the formula in symbols as:

$$XZ = \sqrt{YZ^2 + YX^2}$$

Substitute the values of each leg

$$XZ = \sqrt{4^2 + 7^2}$$

Get the square of each leg

$$XZ = \sqrt{16 + 49}$$

Simplify

$$XZ = \sqrt{65}$$

Use the table for square roots to simplify

$$XZ = 8.062 \text{ or } 8.06$$

Therefore, the measure of XZ or the hypotenuse of $\triangle XYZ$ is 8.06.

Example 4

Find the measure of the hypotenuse of a right triangle whose legs measures 5 cm and 11 cm respectively. Round off answer to the nearest tenths.

Solution

The problem does not show a figure yet we can still solve for the measure of the hypotenuse because the measures of the two legs are given. We represent the hypotenuse as x (you can use any letter of your choice)

We write the formula and

Substitute the values of each leg

$$x = \sqrt{5^2 + 11^2}$$

Get the square of each leg

$$x = \sqrt{25 + 121}$$

Simplify

$$x = \sqrt{146}$$

Use the table for square roots to simplify

$$x = 12.083 \text{ or } 12.1$$

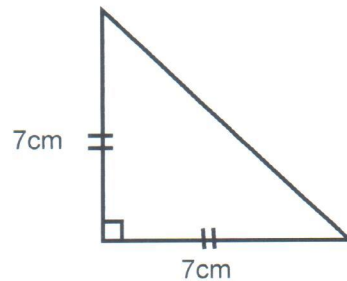
Therefore, the measure of the hypotenuse of the triangle is 12.1 cm.

Example 5

Given an isosceles right triangle, find the measure of the hypotenuse wherein one leg measures 7 cm.

Solution:

Since the given right triangle is an isosceles, it means that the measures of the two legs are congruent. Drawing and labelling the figure it will look like this



We write the formula and represent the unknown as x

Substitute the values of each leg	$x = \sqrt{7^2 + 7^2}$
Get the square of each leg	$x = \sqrt{49 + 49}$
Simplify	$x = \sqrt{98}$
Use the table for square roots to simplify	$x = 9.899$ or 9.90

Therefore, the measure of the hypotenuse of the triangle is 9.90 cm.

Example 6

Find the measure of the hypotenuse of a right triangle wherein the longer leg measures 11 cm while the shorter leg measures 8 less than the measure of the longer leg.

Solution:

Analysing the problem, the measure of the longer leg is 11 cm. The shorter leg measures 8 less than the measure of the longer leg. Therefore we can say that the shorter leg is $11 \text{ cm} - 8 \text{ cm} = 3 \text{ cm}$. Using these measures of legs 11 cm and 3 cm respectively.

We now write the formula as	$x = \sqrt{11^2 + 3^2}$
Get the square of each leg	$x = \sqrt{121 + 9}$
Simplify	$x = \sqrt{130}$
Use the table for square roots to simplify	$x = 11.402$ or 11.40

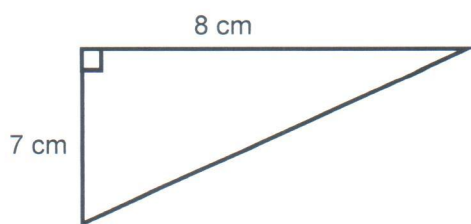
Therefore, the measure of the hypotenuse of the triangle is 11.40 cm.

NOW DO PRACTICE EXERCISE 7

**Practice Exercise 7**

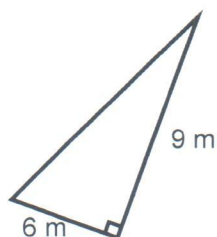
1. Find the measure of the hypotenuse given the following conditions:

a)



Answer: _____

b)



Answer: _____

c) Given Triangle ACB where $AC = 5$ cm and $BC = 9$ cm.

Answer: _____

d) Given a triangle wherein the measure of the shorter leg is 2 cm while the measure of the longer leg is three times the measure of the shorter leg.

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 8: Finding the Adjacent and Opposite Sides



You learnt about what a right triangle is and the Pythagorean Theorem.

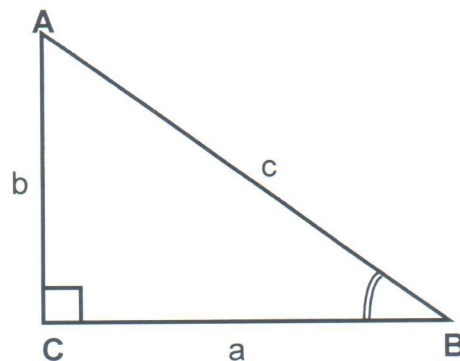


In this lesson, you will:

- define adjacent and opposite sides
- solve for the measures of the adjacent and opposite sides of a right triangle.

In the previous lesson the hypotenuse was highlighted and the other sides were called as legs. The legs of a right triangle may be specifically named as adjacent and opposite sides with respect to a certain angle.

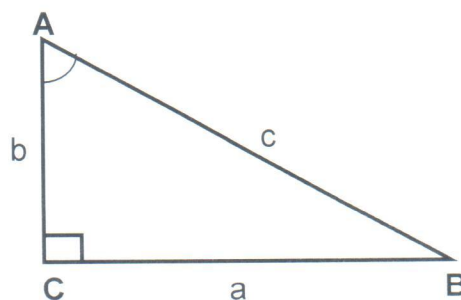
Observe the triangle below



Given $\triangle ACB$, the right angle is $\angle C$, while $\angle A$ and $\angle B$ are the acute angles. **Side c** or $\overline{AB}$ is the hypotenuse while **Side a** or $\overline{CB}$ and **Side b** or $\overline{AC}$ are the legs of the right triangle.

With $\angle B$ as a reference, its adjacent side is **Side a** or $\overline{CB}$. Notice that **Side a** or $\overline{CB}$ is a leg of $\angle B$. Meaning, the adjacent side of a triangle with respect to a given angle is contained or a part of the angle itself.

The adjacent side of an acute angle in a right triangle is a leg contained in the acute angle other than the hypotenuse.



With $\angle A$ as a reference, its adjacent side is **side b** or $\overline{AC}$. Notice that **side b** or $\overline{AC}$ is a leg of $\angle A$.

Now the opposite side with reference to $\angle A$ is **side a** or $\overline{CB}$. Notice that the opposite side of angle is the side representing the same alphabet in lower case.

What does it mean?

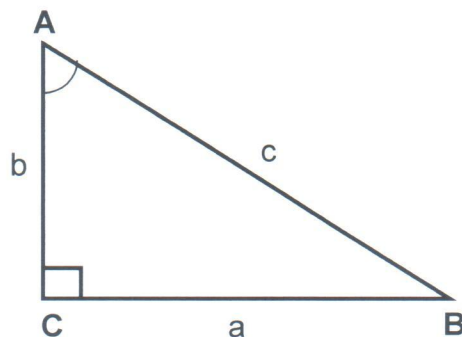
The opposite side of $\angle A$ is **side a**, the opposite side of $\angle B$ is **side b** and the opposite side of the right angle $\angle C$ is **side c** or known as the hypotenuse.

Got it?

Generally, we say that

The opposite side of an acute angle in a right triangle is a side which is not contained in the reference angle. The opposite side intersects the two legs of the reference angle.

Using the same triangle with reference to $\angle A$, its opposite side is side **a** which is not a leg of $\angle A$. Side **a** intersects the legs of $\angle A$ which are side **b** and side **c**.

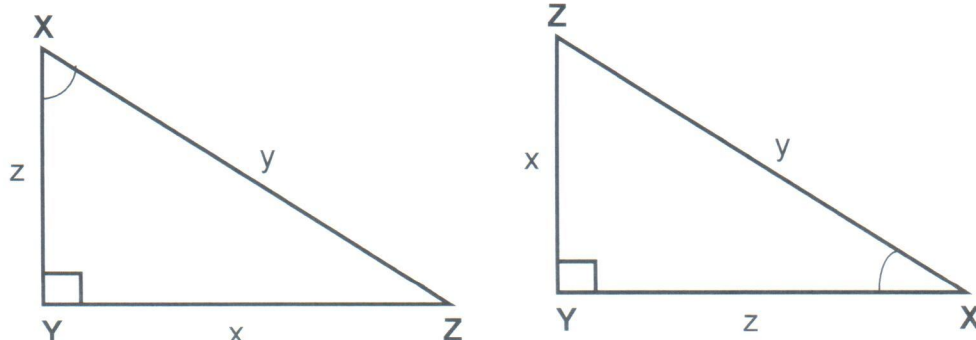


To know the adjacent and opposite sides of an angle in a given triangle, you have to set first a reference angle.

Example 1

Draw and label the $\triangle XYZ$ where Y is the right angle and identify the adjacent and opposite sides with reference to $\angle X$.

Solution:



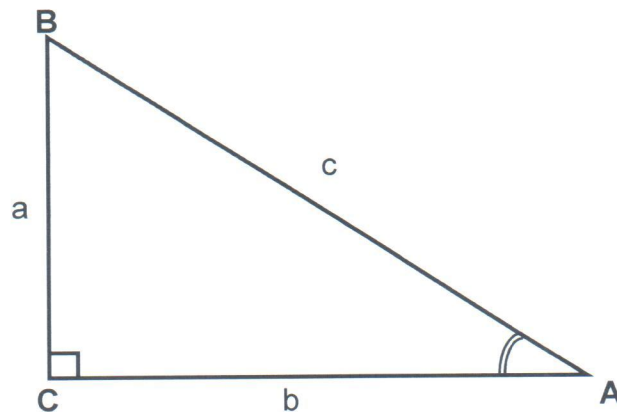
Notice that the triangle maybe drawn in two ways. But both triangles will result to the same adjacent and opposite sides with respect or reference to $\angle X$.

The adjacent side is **side z** or $\overline{YX}$. Notice that in both figures in the previous page, **side z** is contained in $\angle X$.

The opposite side of $\angle X$ is **side x** or $\overline{XY}$. Notice that the opposite side represents the same letter of the reference angle. **Side x** or $\overline{XY}$ is not contained in $\angle X$ and it intersects **side y** or the hypotenuse and **side z** or $\overline{YX}$.

Solving for the Adjacent and Opposite Sides

To solve for the adjacent side (side b or $\overline{AC}$) with respect to $\angle A$, we derive the formula by using the Pythagoras Formula defined by the right triangle ACB as drawn below:



Use the formula represented as
 Transpose a^2 to the left side of the equation
 Interchange the left and right side of the equation
 Get the square roots of both sides of the equation
 Simplify

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 - a^2 &= b^2 \\ b^2 &= c^2 - a^2 \\ \sqrt{b^2} &= \sqrt{c^2 - a^2} \\ b &= \sqrt{c^2 - a^2} \end{aligned}$$

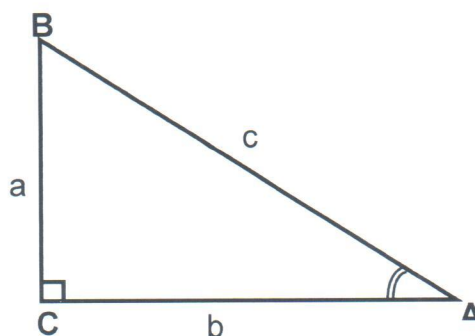
Therefore, we already derived the formula in solving for the adjacent side.

To solve for the measure of the adjacent side given a right triangle ACB wherein C is the right angle, we get the square root of the difference of the squares of the hypotenuse and the opposite side. In symbol we represent the formula as:

$$b = \sqrt{c^2 - a^2}$$

Wherein: **b** is the measure of the adjacent side,
a is the opposite side and
c is the hypotenuse.

To solve for the opposite side (side a or $\overline{BC}$) with respect to $\angle A$, we derive the formula by using the Pythagoras Formula defined by the right triangle ACB as drawn below:



Use the formula represented as
 Transpose b^2 on the left side of the equation
 Interchange the left and right side of the equation
 Get the square roots of both sides of the equation
 Simplify

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 - b^2 &= a^2 \\ a^2 &= c^2 - b^2 \\ \sqrt{a^2} &= \sqrt{c^2 - b^2} \\ a &= \sqrt{c^2 - b^2} \end{aligned}$$

Therefore, we already derived the formula in solving for the adjacent side.

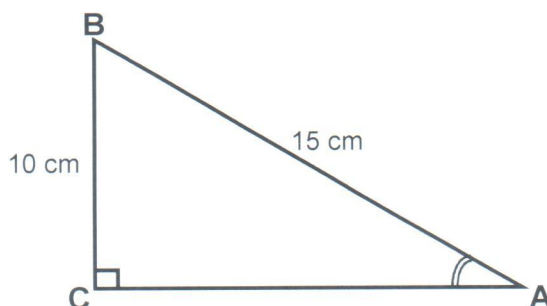
To solve for the measure of the opposite side given a right triangle ACB wherein C is the right angle, we get the square root of the difference of the squares of the hypotenuse and the adjacent side. **In symbol we represent the formula as:**

$$a = \sqrt{c^2 - b^2}$$

wherein a is the measure of the opposite, b is the adjacent side and c is the hypotenuse.

Example 2

Identify the unknown in the figure with respect to the labelled angle then solve for its measure.



Solution:

Since the labelled angle is $\angle A$, the given are side a , which is the opposite side and side c which is the hypotenuse. Therefore, the unknown side is the adjacent side or side b or segment AC .

Identifying the given: $c = 15 \text{ cm}$, $a = 10 \text{ cm}$

Solving for b we use the formula

$$b = \sqrt{c^2 - a^2}$$

Substitute the values of the given

$$b = \sqrt{15^2 - 10^2}$$

Get the squares of the given

$$b = \sqrt{225 - 100}$$

Simplify

$$b = \sqrt{125}$$

Get the square root of 125

$$b = 11.18$$

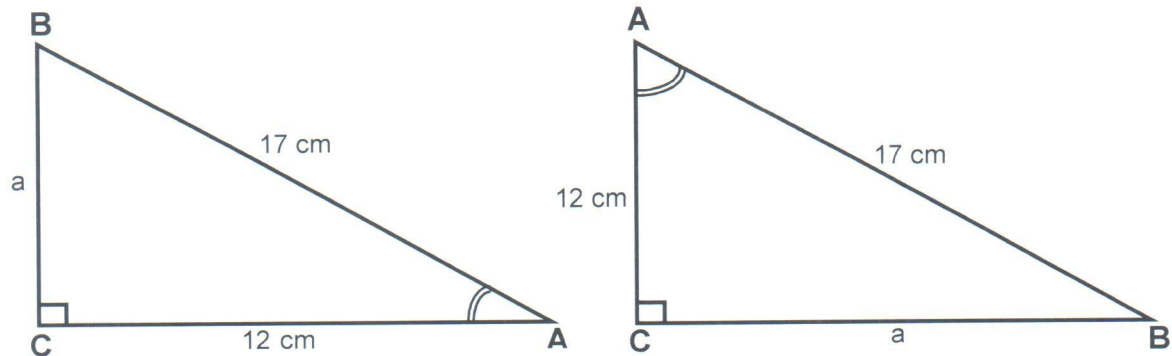
Therefore, the measure of b or the adjacent side is 11.18 cm.

Example 3

Given triangle **ACB** where **C** is the right angle, the measure of the hypotenuse is 17 cm while the measure of the adjacent side with respect to angle **A** is 12 cm. Draw the triangle, label it properly and find the measure of the unknown side of the right triangle.

Solution:

The figure may be drawn in two ways.



The given are the hypotenuse and the adjacent side, therefore, the unknown is the opposite side.

Identifying the given

$$c = 17 \text{ cm}, b = 12 \text{ cm}$$

Solving for a we use the formula

$$a = \sqrt{c^2 - b^2}$$

Substitute the values of the given

$$a = \sqrt{17^2 - 12^2}$$

Get the squares of the given

$$a = \sqrt{289 - 144}$$

Simplify

$$a = \sqrt{145}$$

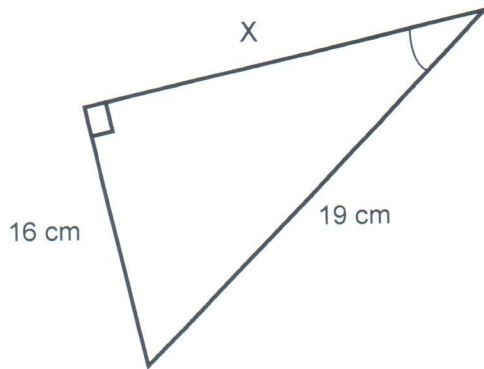
Get the square root of 180

$$a = 2.042 \text{ or } 12.04$$

Therefore, the measure of a or the opposite side is 12.04 cm.

Example 4

Solve for x in the figure.



Solution:

Since the points in the triangle are not labelled, we just identify the given parts with reference to the marked angle. 19 cm is the hypotenuse since it lies opposite the right angle. 16 cm is the opposite side. Therefore the unknown is the adjacent side. With this we use the general idea in solving for the adjacent side, that is, **the measure of the adjacent side (x) is the square root of the difference of the squares of the hypotenuse and the opposite side.**

Identifying the given

hypotenuse = 19 cm, opposite side = 16 cm

Solving for x we use

$$x = \sqrt{19^2 - 16^2}$$

Get the squares of the given

$$b = \sqrt{361 - 256}$$

Simplify

$$b = \sqrt{105}$$

Get the square root of 125

$$b = 10.247 \text{ or } 10.25$$

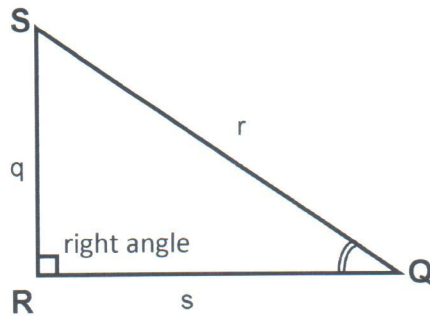
Therefore, the measure of x or the adjacent side is 10.25 cm.

At this point, you can now solve for the measures of the opposite side and adjacent side of a right triangle. Just take note that before you label the given, you have to determine first the reference angle to be used.

NOW DO PRACTICE EXERCISE 8

**Practice Exercise 8**

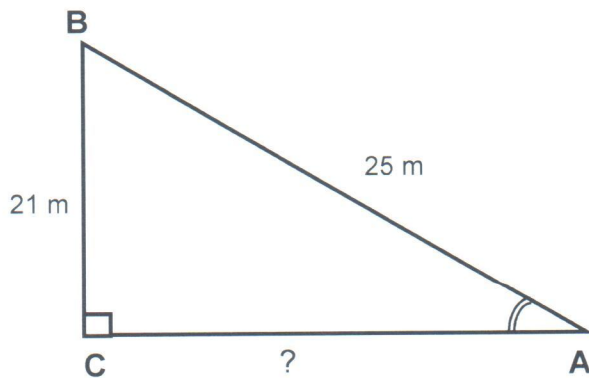
1. Label the right triangle **QRS** where **R** is the right angle. With respect to angle **Q** identify the adjacent and opposite sides.



Adjacent Side: _____
Opposite Side: _____

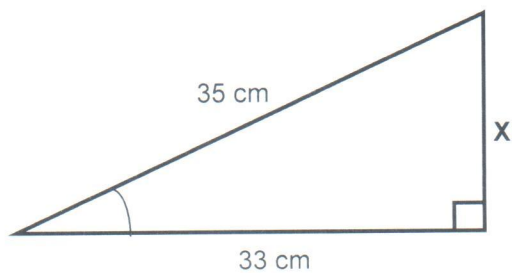
2. Solve the following:

a)



Answer: _____

b)



Answer: _____

- c) Given triangle **CHE** where **H** is the right angle, the measure of the hypotenuse is 35 cm while the measure of the opposite side with respect to angle **C** is 33 cm.

Draw the triangle, label it properly and find the measure of the unknown side of the right triangle.

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 9: Solving Problems Involving Pythagoras' Theorem



You learnt thoroughly in the previous three lessons about the Pythagoras' theorem.



In this lesson, you will:

- identify what is the unknown among the hypotenuse, opposite side and adjacent side of a right triangle in the given practical problem
- solve practical problems using the Pythagoras' Theorem.

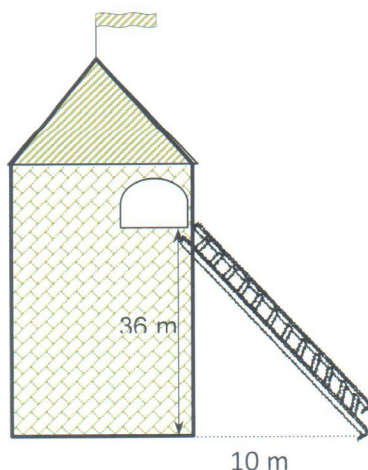
You might be wondering in what way the Pythagoras' Theorem can be of help to you in your everyday life.

Have you ever dreamt of becoming an engineer someday? How about an architect?

This lesson will not only help you prepare for the above mentioned careers in the future, it will also help you explore practical uses of the theorem in our daily lives.

Example 1

A clown is locked in a tower of a Castle in an amusement park. You have volunteered to rescue him. If the tower window is 36 metres above the ground and you must place your ladder 10 metres from the base of the castle, what is the shortest length of a ladder you will need to reach the tower window?



Solution:

Since the unknown is the measure of the ladder (hypotenuse) we use the formula

$$c = \sqrt{36^2 + 10^2}$$

We simplify:

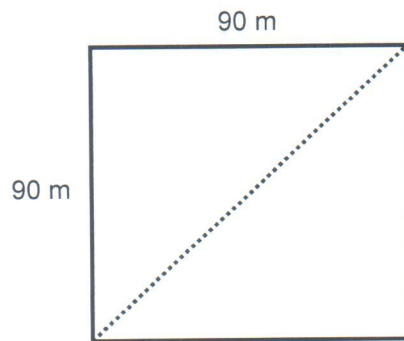
$$c = \sqrt{1296 + 100}$$

$$c = 37.36 \text{ metres}$$

Therefore, the shortest length of a ladder needed to reach the tower window is 37.36 m.

Example 2

A baseball diamond is a square with sides of 90 m. What is the shortest distance, to the *nearest tenth* of a foot, between home plate and second base?



Solution:

Since the figure is square, the shortest distance must be its diagonal which is the hypotenuse of the triangle, we use the formula

$$c = \sqrt{90^2 + 90^2}$$

We simplify:

$$c = \sqrt{8100 + 8100}$$

$$c = \sqrt{16200}$$

$$c = 127.3 \text{ m}$$

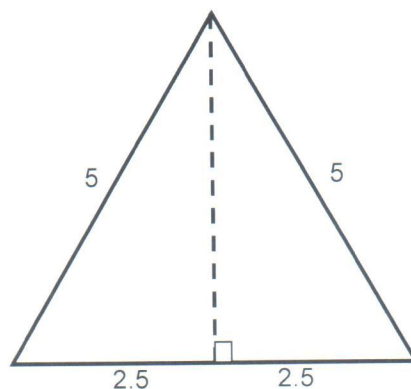
Therefore, the shortest length between home plate and second base is 127.3 m.

Example 3

Using the Pythagorean Theorem, find the area of an equilateral triangle whose side measures 5 units. Find the area to the *nearest tenth* of a square unit.

Solution:

Draw the figure.



Before we can solve for the area of the triangle, we need to solve first for the height of the triangle. The height is the opposite side of the right triangle.

We use the formula:

$$x = \sqrt{5^2 - 2.5^2}$$

Simplify:

$$x = \sqrt{25 - 6.25}$$

$$x = \sqrt{18.75}$$

$$x = 4.33$$

Since the height of the triangle is 4.33 and its base measures 5 units, we solve the area using the formula $A = \frac{1}{2}bh$.

Substitute the values in the formula:

$$A = \frac{1}{2}(5)(4.33)$$

Simplify:

$$A = \frac{1}{2}(21.65)$$

$$A = 10.825$$

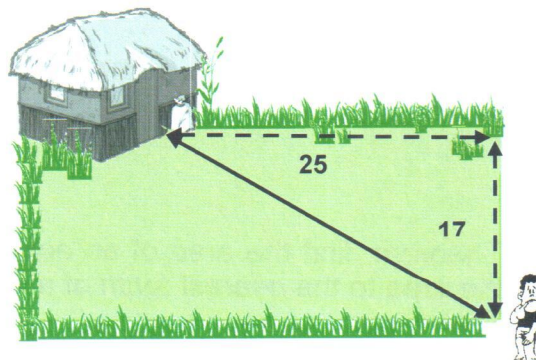
Round off to the nearest tenth:

$$A = 10.8$$

Therefore, the area of the equilateral triangle is 10.8 units.

Example 4

Joshua regularly takes a short-cut across their lawn instead of walking on the sidewalk on his way home from school. How much distance is saved by Joshua cutting across the lawn?



Solution:

We use the formula:

$$x = \sqrt{25^2 + 17^2}$$

Simplify:

$$x = \sqrt{625 + 289}$$

$$x = \sqrt{914}$$

$$x = 30.23$$

If Josh will walk along the path he will have to walk $25\text{m} + 17\text{m} = 42\text{ m}$. If he cuts across the lawn he will only walk 30.23 m.

Therefore Joshua saved 11.77 m ($42\text{m} - 30.23\text{m}$) by cutting across the lawn.

NOW DO PRACTICE EXERCISE 9

**Practice Exercise 9**

Solve the following word problems:

- 1) A ship travelled 13.5 km due north and travelled 8 km due east. How far is the ship from the starting point?

Answer: _____

- 2) Christen tied her kite on a peg on the ground, then she went exactly down the kite which is 15 m away from the peg. If the kite used a 25 m string, how high is the kite?

Answer: _____

- 3) A rectangle with a length of 13 cm and a width of 8 cm is cut along its diagonal. How long is its diagonal?

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.
--

TOPIC 2: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- The **square of a number** is the product when the number is multiplied by itself.
- In symbols we can say that the square of a number n is $n \times n$ or n^2 .
- The **square root of a number** refers to a number which to be multiplied by itself to get the given number.
- In symbols, we can write the square root of a number n as $\sqrt{n}$.
- The square root of perfect square number is a whole number.
- A **Right Triangle** contains one right angle and two acute angles.
- The **hypotenuse** is the longest side of a right triangle and it is located opposite the right angle.
- **Pythagoras Theorem** is defined as "the square of the hypotenuse is equal to the sum of the squares of the two legs" denoted by

$$c^2 = a^2 + b^2$$

- To solve for the measure of the hypotenuse given a right triangle ACB wherein C is the right angle, we get the square root of the sum of the squares of the measures of the two legs. In symbol we represent the formula as

$$c = \sqrt{a^2 + b^2}$$

wherein c is the measure of the hypotenuse and a & b are the measures of the legs respectively.

- To solve for the measure of the adjacent side given a right triangle ACB wherein C is the right angle, we get the square root of the difference of the squares of the hypotenuse and the opposite side. In symbol we represent the formula as

$$b = \sqrt{c^2 - a^2}$$

wherein b is the measure of the adjacent side, a is the opposite side and c is the hypotenuse.

- To solve for the measure of the opposite side given a right triangle ACB wherein C is the right angle, we get the square root of the difference of the squares of the hypotenuse and the adjacent side. In symbol we represent the formula as

$$a = \sqrt{c^2 - b^2}$$

wherein a is the measure of the opposite, b is the adjacent side and c is the hypotenuse.

REVISE LESSON 5 – 9. THEN DO TOPIC TEST 2 IN ASSIGNMENT BOOK 5.

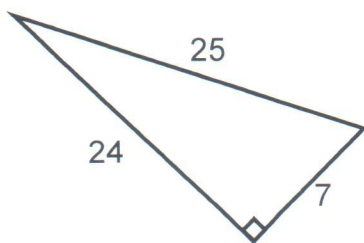
ANSWERS TO PRACTICE EXERCISES 5-9

Practice Exercise 5

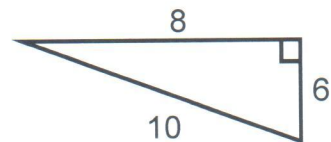
1. a) 81
b) 144
c) 0.81
d) 625
e) 1.44
 2. a) 5.83
b) 11
c) 9.27
d) 11.58
-

Practice Exercise 6

1. a)

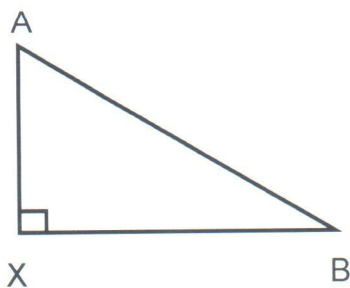


b)



2. a) No. $100 \neq 81$ (100 is not equal to 81)
b) Yes. $56.25 = 56.25$ (56.25 is equal to 56.25)

3.



hypotenuse: $\overline{AB}$
legs: $\overline{AX}$ and $\overline{BX}$

Practice Exercise 7

1. a) 10.63 cm
b) 10.82 m
c) 10.30 cm
d) 6.32 cm

Practice Exercise 8

1. Adjacent Side : side s or $\overline{RQ}$
Opposite Side: side q or $\overline{RS}$
 2. a) 13.56
b) 11.66 cm
c) 11.66 cm
-

Practice Exercise 9

- 1) 15.69 km
 - 2) 20 m
 - 3) 15.26 cm
-

END OF TOPIC 2

TOPIC 3

POLYGONS

Lesson 10: Quadrilaterals

Lesson 11: The Interior Angle Sum of a Polygon

Lesson 12: Deductive Reasoning and Pythagoras' Theorem

Lesson 13: Circle Theorems 1 - 3

Lesson 14: Tangent Theorems 1-3

Lesson 15: General Problems Involving Circles

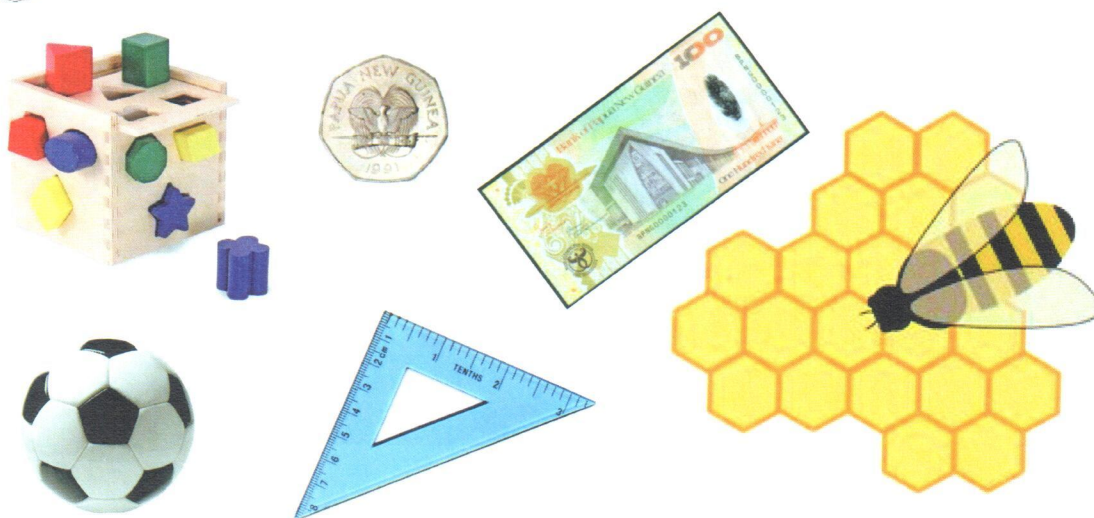
Lesson 16: Circle Problems Associated with Pythagoras Theorem

TOPIC 3: POLYGONS AND THE CIRCLES

Introduction



Observe the objects below.
What do you think makes them unique and more eye-catching?



Great observation if you say that they are made up of different shapes.

From the mind building toys you had in your childhood, to your favourite sports in the field, to the currency we use every day and to the great art Mother Nature gives us, we see a lot of different shapes that make up different objects.

You first learned them as shapes, but this topic will lead you to a more concrete term that defines them as polygons. The topic will also lead you to the different properties of polygons and how these properties will be used to solve practical problems we may encounter in our daily living.

Reasoning will also be discussed.

Lesson 10: Quadrilaterals

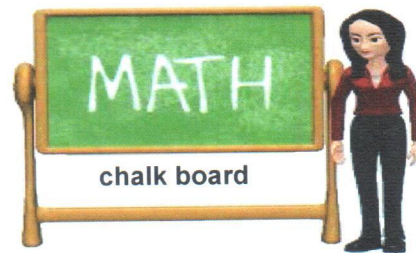
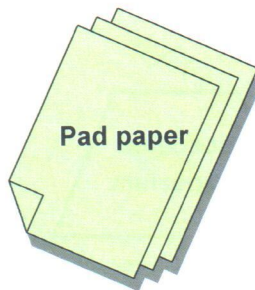
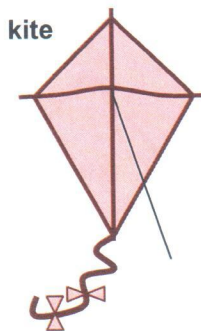


You have learnt something about triangles in the previous topic.

In this lesson, you will:

- define quadrilateral and its different types
- discuss the angle properties of parallelogram and special quadrilaterals.

What do the following objects have in common?



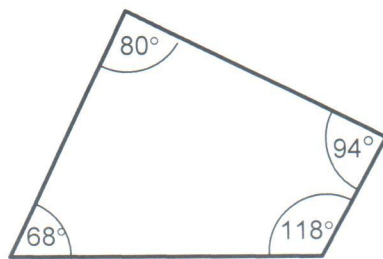
Did you notice that they have the same number of sides?
How about their corners?

Good observation if you say that all these figures have 4 sides (edges) and 4 vertices (corners). The shape of these objects is known as quadrilateral.

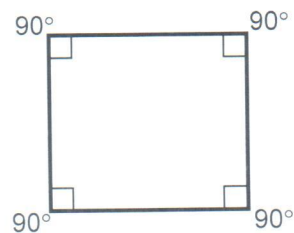
A quadrilateral is a four-sided polygon. The following are the general properties of every quadrilateral:

- **Four sides (edges)**
- **Four vertices (corners)**
- **The interior angles add up to 360 degrees**

Try to draw any quadrilateral, and measure the angles in each corner using a protractor.



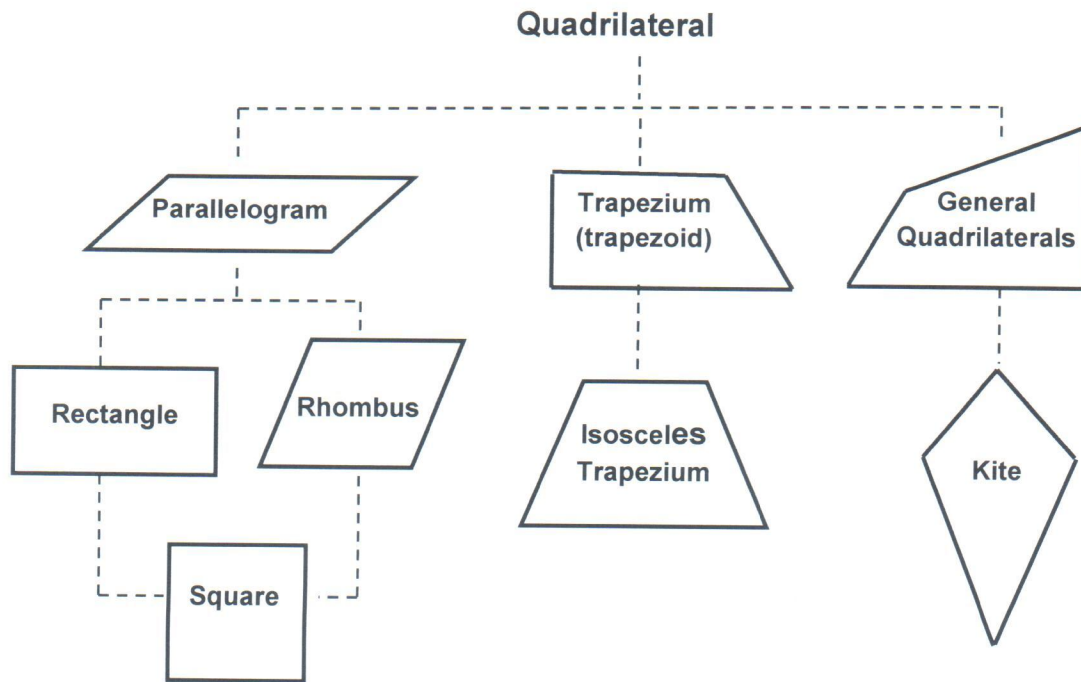
$$68^\circ + 118^\circ + 94^\circ + 80^\circ = 360^\circ$$



$$4 \times 90^\circ = 360^\circ$$

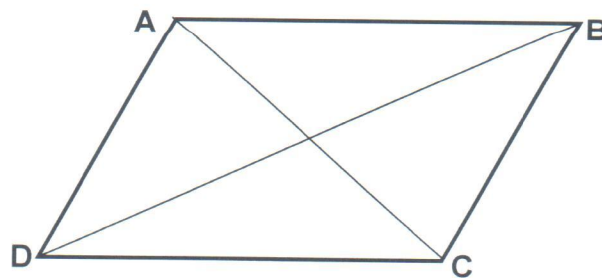
Types of Quadrilaterals

There are several types of quadrilateral with different or unique properties. The following diagram illustrates the types of quadrilaterals and their relationship with each other.



Parallelogram is a quadrilateral with **two pairs of parallel sides** and **opposite sides congruent**.

In the figure below $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. The opposite sides are also congruent, $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. $\overline{AC}$ and $\overline{BD}$ are the diagonals.



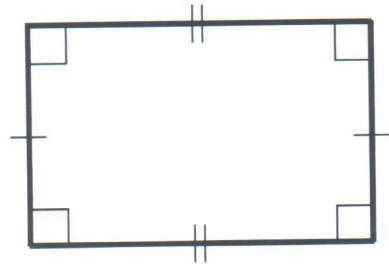
Properties of all parallelograms

- 1) Opposite sides are parallel,
- 2) Opposite angles are congruent,
- 3) Opposite sides are congruent,
- 4) Consecutive angles are supplementary, and
- 5) Diagonals bisect each other.

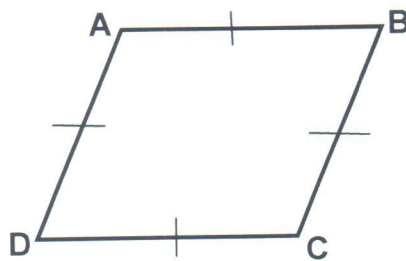
Rectangle is a parallelogram with 4 right angles.

Special properties of a rectangle

- 1) All four angles of a rectangle are right angles.
- 2) The diagonals of a rectangle are congruent.



Rhombus is a parallelogram with 4 equal sides.



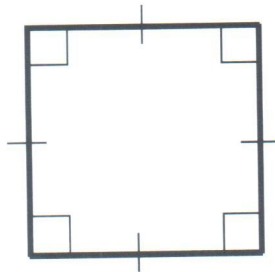
Notice that since the figures above are also parallelograms $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. The only difference is that all four sides are congruent.

Special properties of a rhombus

- 1) All four sides are congruent.
- 2) The diagonals of a rhombus bisect each other.

Square is both a rhombus and a rectangle. It is a rhombus because it has 4 equal sides. It is a rectangle because it has 4 right angles.

The figure below is an example of a square.

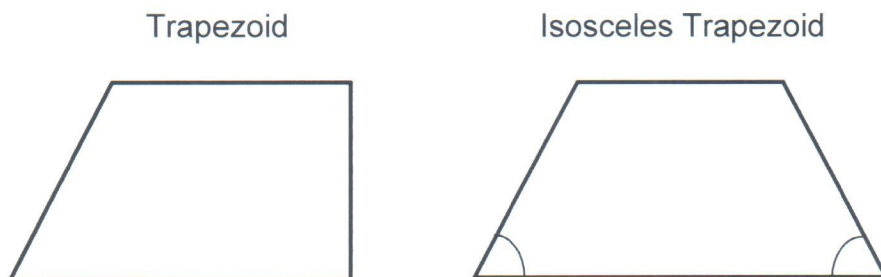


The square has all the properties of a rhombus and a rectangle.

- 1) All four angles of a rectangle are right angles.
- 2) The diagonals of a rectangle are congruent.
- 3) All four sides are congruent.
- 4) The diagonals of a rhombus bisect each other.

Trapezium (called a **trapezoid** in the US) has a pair of opposite sides parallel.

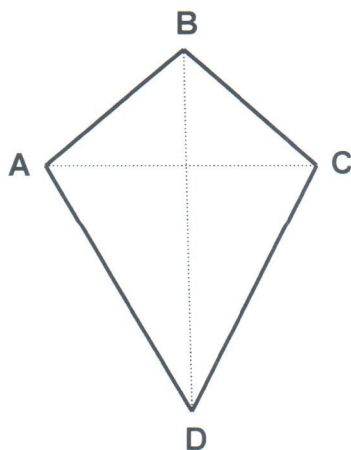
It is called an **isosceles trapezium** or **isosceles trapezoid** if the sides that are not parallel are equal in length and both angles coming from a parallel side are equal, as shown.



General Quadrilaterals are any quadrilaterals with no parallel sides. The most common of these is the **KITE** with special properties.

Properties of a kite

- 1) Two pairs of adjacent sides of a kite are equal in length
- 2) One pair of opposite angles (the ones that are between the sides of unequal length) are equal in size.
- 3) One diagonal bisects the other.
- 4) Diagonals intersect at right angles.



In the figure above, $\overline{AB}$ and $\overline{BC}$ are congruent, $\angle A$ and $\angle C$ are the pair of opposite angles which are congruent. Diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other, meaning they divide each other into two congruent segments. And lastly, the intersection of the two diagonals is a right angle.

Now that you have mastered the different properties and characteristics of different quadrilaterals, you are now ready to answer the following practice exercise.

NOW DO PRACTICE EXERCISE 10



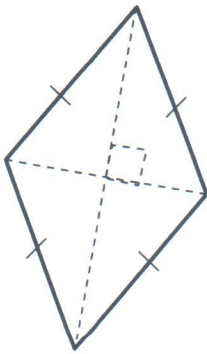
Practice Exercise 10

1. Write TRUE if the statement is correct, otherwise write FALSE.

- a) A parallelogram is quadrilateral. _____
- b) A kite has four congruent sides. _____
- c) A rectangle has 4 right angles. _____
- d) A square is both a rectangle and a rhombus. _____
- e) A trapezoid has two pairs of opposite sides parallel. _____
- f) A general quadrilateral has one or two pairs of parallel sides. _____
- g) The measures of the angles of any quadrilateral add up to 360° . _____
- h) A trapezoid is also known as a trapezium. _____
- i) The diagonals of a square are congruent. _____
- j) The diagonals of a kite are perpendicular with each other. _____

2. Identify the quadrilateral drawn below:

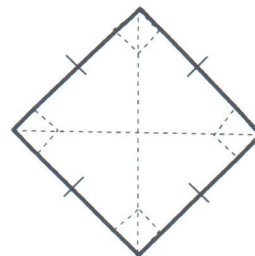
a)



b)

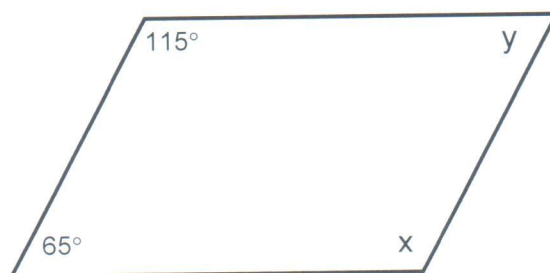


c)



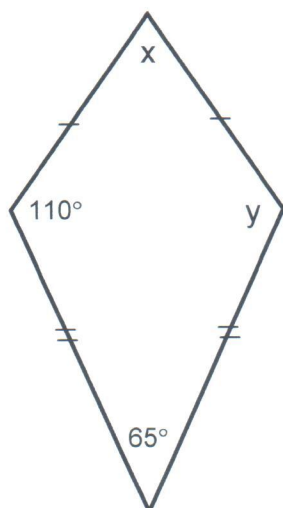
3. Find the value of the pronumerals (x and y) in the following diagram. Give reasons for your answer.

a)



Answer: $x =$ _____
 $y =$ _____

b)



Answer: $x =$ _____
 $y =$ _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 11: The Interior Angle Sum of a Polygon



You learnt about quadrilaterals in the previous lesson.



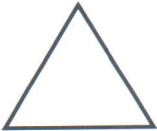
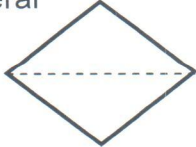
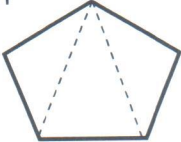
In this lesson, you will:

- calculate the Interior Angle Sum of a Polygon
- solve for the measure of an angle of a regular polygon given the angle sum.

In the previous lessons, we have discussed that the sum of all angles in a triangle is 180° while the angle sum in every quadrilateral is 360° discussed in Lesson 10. What if you were asked about the angle sum of other polygons such as pentagon, hexagon, octagon and nonagon?

This lesson will help you determine the angle sum of any polygon.

Try to observe the following illustrations:

Figure	Number of Sides	Number of triangles formed from 1 vertex	Interior Angle Sum
Triangle 	3	1	180°
Quadrilateral 	4	2	360
Pentagon 	5	3	?

What do you think is the interior angle sum of the pentagon?

Observe how the interior angle sum relates with the number of triangles formed and the number of sides of the polygon.

Did you notice that the angle sum could be determined by multiplying the number of triangles formed from one vertex by 180° ?

Therefore, can we say that the interior angle sum of a pentagon is $3(180^\circ) = 540^\circ$?

But what if you were given a 20 sided polygon? How can we determine the number of triangles from one vertex?

Drawing the figure is helpful but it takes a lot of time.

If you notice on the table in the previous page that in a triangle the number of triangles formed from 1 vertex is 1, 2 for a quadrilateral and 3 for a pentagon.

There is a common difference of 2!

With that we can conclude that the number of triangles formed from one vertex is simply the number of sides (n) take away 2. In symbol we say $n - 2$.

Combining all the observations above, we say that:

The **Interior Angle Sum of a Polygon** can be solved by multiplying the number of triangles formed from one vertex with 180° . It can also be represented by the notation

$$S = (n - 2)180^\circ$$

Where: S = interior angle sum

n = number of sides of a polygon

Example 1

What is the Interior angle sum of a heptagon?

Solution:

A heptagon is a polygon with 7 sides. Using the formula $s = (n - 2)180^\circ$

$$S = (7 - 2) 180^\circ$$

$$S = (5)(180^\circ)$$

$$S = 900^\circ$$

Therefore the interior angle sum of a heptagon is 900° .

Example 2

What is the interior angle sum of a decagon?

Solution:

A decagon is a polygon with 10 sides. Using the formula $s = (n-2)180^\circ$

$$s = (10 - 2) 180^\circ$$

$$s = (8)(180^\circ)$$

$$s = 1440^\circ$$

Therefore the interior angle sum of a decagon is 1440° .

Example 3

What is the measure of one angle of a regular hexagon?

Solution:

A hexagon is a polygon with 6 sides. Using the formula $S = (n-2)180^\circ$

$$S = (6-2) 180^\circ$$

$$S = (4) (180^\circ)$$

$$S = 720^\circ$$

But wait!

The problem requires for the measure of one angle of a regular hexagon. Since the hexagon is regular, meaning all angles are congruent, we divide the interior angle sum by the number of sides which is 6.

$$720^\circ \div 6 = 120^\circ$$

Therefore the measure of one angle of a regular hexagon is 120° .

Example 4

What is the measure of one angle of a regular 12-sided polygon?

Solution:

Since we now know that the polygon has 12 sides we use the formula $S = (n-2)180^\circ$

$$S = (12-2) 180^\circ$$

$$S = (10) (180^\circ)$$

$$S = 1800^\circ$$

But wait!

The problem requires for the measure of one angle of a regular 12-sided polygon. We will divide the interior angle sum by the number of sides which is 12.

$$1800^\circ \div 12 = 150^\circ$$

Therefore the measure of one angle of a 12-sided polygon is 150° .

NOW DO PRACTICE EXERCISE 11

**Practice Exercise 11**

1. Find the interior angle sum of the following polygons.

a. Nonagon

Answer: _____

b. Octagon

Answer: _____

c. Dodecagon

Answer: _____

d. 15-sided polygon

Answer: _____

2. Find the measure of one angle in the following regular polygons.

a. Hexagon

Answer: _____

b. Octagon

Answer: _____

c. 13-sided polygon

Answer: _____

d. 16-sided polygon

Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
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Lesson 12: Deductive Reasoning



You learnt about the Pythagoras' Theorem and its applications in the previous topics.



In this lesson, you will:

- define deductive reasoning
- use deductive reasoning to solve problems involving triangles and quadrilaterals.

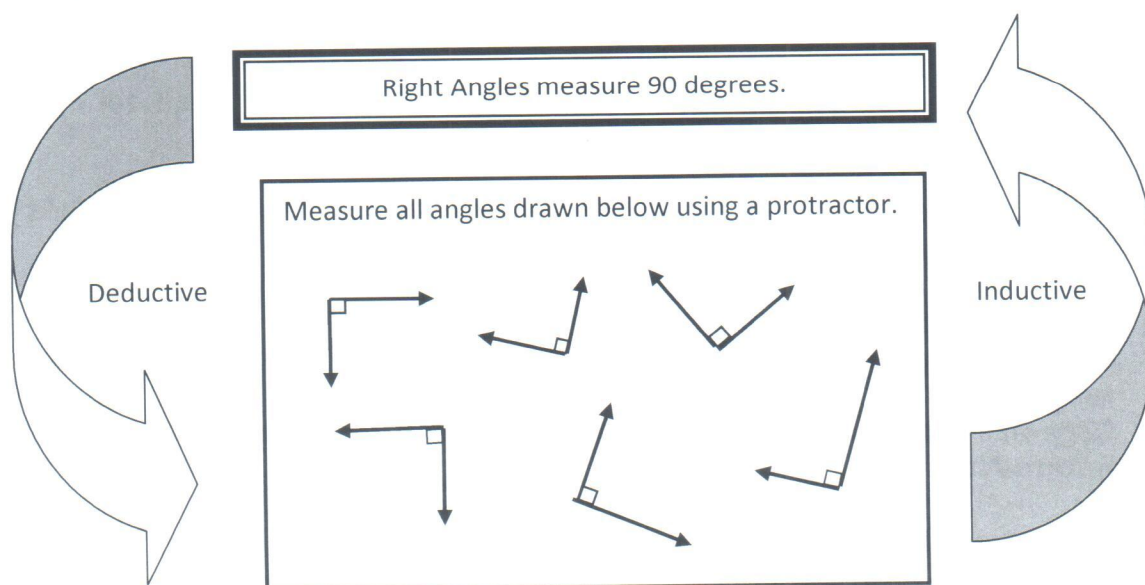
Deductive reasoning is a kind of reasoning from a general concept down to specific one. Others call it as “thinking it out” which refers to the method of solving problems by applying step by step properties.

When we arrive at a conclusion using facts, definitions, rules or properties, it is called **Deductive Reasoning**.

A generalized concept will be proven using a series of examples. This also involves showing specific examples to prove a generalized concept.

In Mathematics, deductive reasoning is widely used especially in proving theorems and in explaining how a formula was derived.

The figure below shows how deductive reasoning is carried out. A generalization is given and specific cases or examples will be used to prove the first. Inductive reasoning is the reverse process, wherein from specific examples or cases you will come up with a generalized idea.



Deductive Reasoning is a form of reasoning by which each conclusion follows from the previous one; an argument is built by conclusions that progress towards a final statement.

Deductive reasoning can be shown by writing **Two-Column Proof**.

What is a Proof?

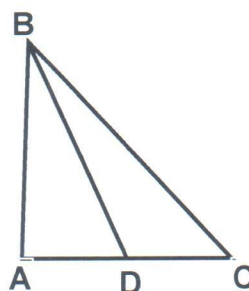
A **proof** is a written account of the complete thought process that is used to reach a conclusion. Each step of the process is supported by a theorem, postulate or definition verifying why the step is possible.

If you think about the numerical problems you are used to solving in Geometry, you will realize that your mind often does a “fast forward” through some of the logical steps needed to reach a valid answer. In other words, you quickly “go right to the answer”.

Check out the numerical problem:

Given: $\triangle ABD$, with $\overline{BD}$ bisecting $\overline{AC}$
 $\overline{AD} = 6$

Find the length of $\overline{DC}$.



Solution:

Since $\overline{BD}$ bisects $\overline{AC}$, D is the midpoint of $\overline{AC}$, forming two congruent segments whose measures are equal. $\overline{AD} = 6$, then $\overline{DC} = 6$ as well.

You probably arrived at the answer of 6 long before you finished reading the explanation of the answer. Right?

When developing a proof, we must be careful to include **all** of the steps that led to our answer. We cannot “fast-forward” over steps when writing a proof.

A proof requires that you file all the little steps that you mentally “fast-forward” through the numerical problem. Mathematical proof takes an accepted set of facts and properties to demonstrate something to be true.

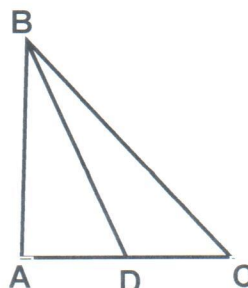
In a two column proof, statements are made in the left column and justifications for those statements are on the right.

1. **Begin with the given information.**
2. **Work through the diagram to determine whether the conclusion can be reached.**
3. **Organize the steps carefully as in the example below, including the given information.**

Take for example the proof of the same problem above.

Given: $\triangle ABD$, with $\overline{BD}$ bisecting $\overline{AC}$
 $\overline{AD} = 6$

Prove: $\overline{DC} = 6$



Solution:

Statements	Reason
1. $\overline{BD}$ bisects $\overline{AC}$, $\overline{AD} = 6$	1. Given
2. D is the midpoint of $\overline{AC}$	2. Segment bisector intersects a segment at its midpoint
3. $\overline{AD} \cong \overline{DC}$	3. Midpoint divides a segment into 2 congruent parts.
4. $\overline{AD} = \overline{DC}$	4. Congruent segments have equal measures.
5. $\overline{DC} = 6$	5. Substitution

Here are solved examples on Deductive Reasoning.

Example 1

Using deductive reasoning prove that $(2 \times 4) \times 5 = (5 \times 4) \times 2$.

Solution:

Statements	Reason
1. $(2 \times 4) \times 5 = 5 \times (2 \times 4)$ $8 \times 5 = 5 \times 8$ $40 = 40$	1. Commutative Property of Multiplication (The order of the number does not affect the product).
2. $(2 \times 4) \times 5 = 5 \times (4 \times 2)$ $8 \times 5 = 5 \times 8$ $40 = 40$	2. Again, Commutative Property of Multiplication
3. $(2 \times 4) \times 5 = (5 \times 4) \times 2$ $8 \times 5 = 20 \times 2$ $40 = 40$	3. Associative Property of Multiplication. (The grouping of the numbers does not affect the answer).

Example 2

Given: $2x + 5 = 17$ prove that $x = 6$

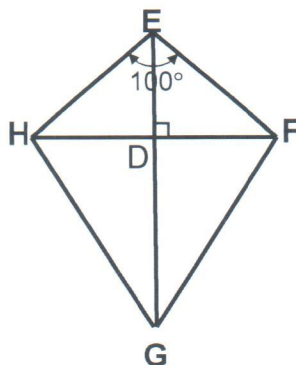
Statements	Reason
2. $2x + 5 = 17$	1. Given
2. $2x = 17 - 5$ $2x = 12$	2. Subtraction Property of Equality (Equals may subtracted from sides of the equation)
3. $x = 6$	3. Division Property of Equality (Divide both sides of the equation by 2.)

In examples 1 and 2, deductive reasoning was used to prove something. Algebraic examples were used to illustrate how deductive reasoning is used.

In the next example, deductive reasoning is use to find the measure of an unknown angle by applying a step by step properties.

Example 3

Given: Kite **EFGH** with $\angle DHG = 75^\circ$, $\angle FEH = 100^\circ$, $\overline{DE} = 3$ cm, $\overline{DF} = 5$ cm and $\overline{FG} = 15$ cm.



Use deductive reasoning, to find the measures of:

- a) $\angle DGH$
- b) $\angle EFH$

Solution:

a) $\angle DGH$

Statement	Reason
1. $\angle DGH + \angle GDH + \angle DHG = 180^\circ$	1. Angle sum of a triangle
2. $\angle DGH = 180^\circ - \angle GDH - \angle DHG$	2. Subtraction property of Equality (SPE)
3. $\angle DHG = 75^\circ$	3. Given
4. $\angle GDH = 90^\circ$	4. Diagonals of a kite are perpendicular therefore, they form right angles
5. $\angle DGH = 180^\circ - 90^\circ - 75^\circ = 15^\circ$	5. Substitution (Substitute 3 & 4 in 2)

Therefore, the measure of $\angle DGH$ is 15° .

b) $\angle EFH$

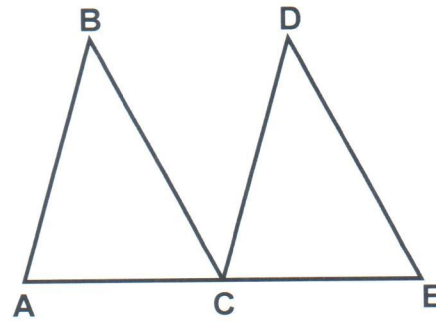
Statement	Reason
1. $\angle EFH + \angle FHE + \angle FEH = 180^\circ$	1. Angle sum of a triangle
2. $\angle FEH = 100^\circ$	2. Given
3. $\angle DFH + \angle FHE + 100 = 180^\circ$	3. Substitution (substitute 2 in 1)
4. $FE = EH$	4. Kite property
5. $\triangle HEF$ is an isosceles $\triangle$	5. Definition of an isosceles $\triangle$
6. $\angle EFH \cong \angle FHE$	6. Base angles of isosceles $\triangle$ are $\cong$
7. $2\angle EFH + 100 = 180^\circ$	7. Substitution
8. $2\angle EFH = 180^\circ - 100^\circ = 80^\circ$	8. Subtraction Property of Equality
9. $\angle EFH = 40^\circ$	9. Division Property of Equality

Therefore, the measure of $\angle EFH$ is 40° .

Example 4

Given: C is the midpoint of segment AE.
 AB and CD are parallel.
 $\angle B$ and $\angle D$ are congruent.

Prove: $BC = DE$



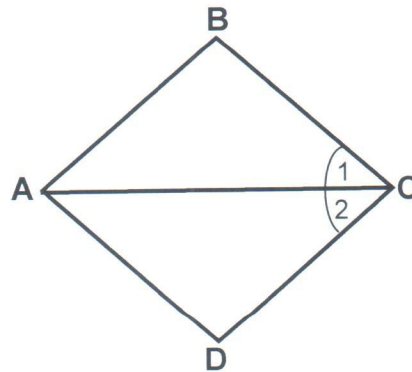
Solution:

Statement	Reason
1. C is the midpoint of segment AE.	1. Given
2. $AC = CE$	2. Definition of midpoint.
3. AB and CD are parallel.	3. Given
4. $\angle ECD \cong \angle CAB$	4. Corresponding angles are equal.
5. $\angle B \cong \angle D$	5. Base angles of isosceles Δ are $\cong$
6. $\Delta ABC \cong \Delta CDE$	6. SAA Congruence
7. $BC = DE$	7. Corresponding parts of Congruent Triangles are congruent. (CPCTC)

Example 5

Given: $BC = CD$
 AC bisects $\angle BCD$

Prove: $\Delta ABC \cong \Delta ADC$



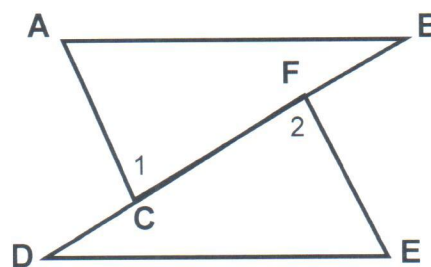
Solution:

Statement	Reason
1. $BC = CD$.	1. Given
2. $\angle 1 = \angle 2$	2. An angle bisector a ray whose endpoints is the vertex of the angle and divides the angle into two equal angles.
3. $AC = AC$	3. Reflexive Property (Identity). Any magnitude is equal to itself.
4. $\Delta ABC \cong \Delta ADC$	4. SAS Congruency.

Example 6

Given: $\overline{AC} \perp \overline{DB}$, $\overline{EF} \perp \overline{DB}$
 $\overline{AC} = \overline{EF}$
 $\angle A = \angle E$

Prove: $\angle B = \angle D$



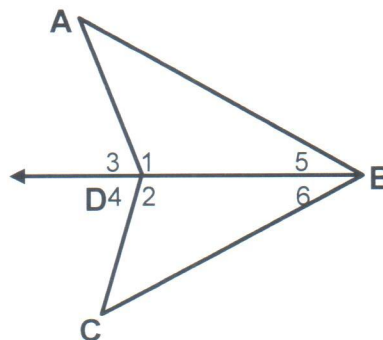
Solution:

Statements	Reasons
1. $\overline{AC} \perp \overline{DB}$, $\overline{EF} \perp \overline{DB}$	1. Given
2. $\angle 1$ and $\angle 2$ are right angles	2. Perpendicular lines meet to form right angles.
3. $\angle 1 = \angle 2$	3. Right angles are equal.
4. $\angle A = \angle E$	4. Given.
5. $\triangle ABC \cong \triangle EDF$	5. ASA Congruency (if two angles and the included side of one triangle are congruent to the corresponding parts of a second triangle, the triangles are congruent).
6. $\angle B = \angle D$	6. Corresponding parts of Congruent Triangles are congruent. (CPCTC)

Example 7

Given: Line segment $\overline{BD}$
 $\overline{AD} \cong \overline{DC}$
 $\angle 3 = \angle 4$

Prove: $\overline{BD}$ bisects $\angle ABC$



Solution:

Statement	Reason
1. Line segment $\overline{BD}$ $\overline{AD} \cong \overline{DC}$ $\angle 3 = \angle 4$	1. Given
2. $\angle 1$ is a supplement of $\angle 3$ $\angle 2$ is a supplement of $\angle 4$	2. Two $\angle$ s in forming linear pair are supplementary.
3. $\angle 1 = \angle 2$	3. Supplements of equal $\angle$ s are equal.
4. $\overline{BD} = \overline{BD}$	4. Reflexive property. (Identity) a quantity is equal to itself
5. $\triangle ABD \cong \triangle CBD$	5. SAS congruency
6. $\angle 3 = \angle 4$	6. Corresponding parts of congruent triangles are congruent (CPCTC).

NOW DO PRACTICE EXERCISE 12



Practice Exercise 12

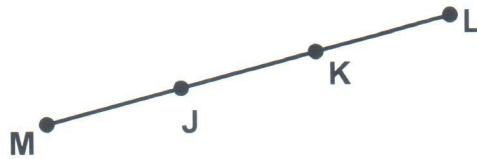
1. Justify each statement with a property from Algebra.

- a. If $6x - 7 = 29$, then $6x = 36$
- b. If $6x = 36$, then $x = 6$
- c. If $x - 3 = 5$ and $x = 8$, then $8 - 3 = 5$

2. Complete the following proof using deductive reasoning.

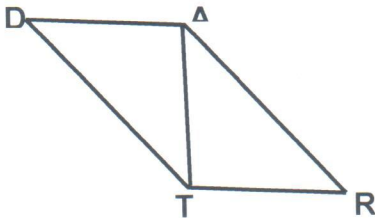
- a. Given: J is the midpoint of $\overline{MK}$
K is the midpoint of $\overline{JL}$

Prove: $\overline{MJ} \cong \overline{KL}$



Statement	Reason
1. J is the midpoint of $\overline{MK}$	1. _____
2. _____	2. Given
3. $\overline{MJ} \cong \overline{JK}$	3. _____
4. _____	4. Definition of midpoint
5. _____	5. _____

b.

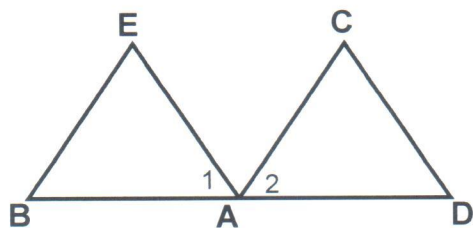


Given: $\triangle GES \cong \triangle STG$
 $\angle ESA \cong \angle TGA$

Prove: $\overline{SA} \cong \overline{GA}$

Statement	Reason
1. $\triangle GES \cong \triangle STG$	1. _____
2. _____	2. CPCTC
3. _____	3. Given
4. _____	4. Identity
5. $\triangle ASE \cong \triangle AGT$	5. _____
6. _____	6. CPCTC

3. Write the two column proof for the diagram below using deductive reasoning.



Given: $\angle BAC \cong \angle DAE$

$\overline{AE} \cong \overline{AC}$

A is a midpoint of $\overline{BD}$

Prove: $\triangle BEA \cong \triangle DCA$

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 13: Circle Theorems 1-3



You learnt something about circles in your previous Mathematics.



In this lesson, you will:

- name and define each part of the circle
- identify and explain circle theorems 1-3.

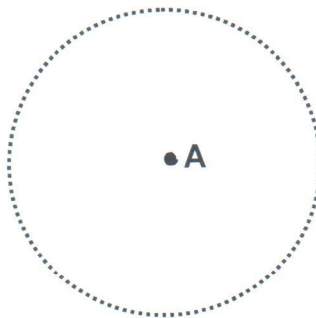
The Circle

Is circle a polygon?

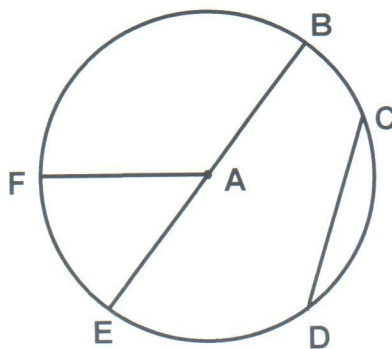
No it is not. A circle is plane figure but not a polygon because it does not have sides.

A **circle** is a set of points equidistant to a fixed point called the **centre**. The center of a circle is named using a capital letter.

The figure below shows circle A or can be written in symbol as $\odot A$ (read as circle A). This means that the centre of the circle is at point A.



The following are the parts of a circle.



Diameter is a segment passing through the centre of the circle and its endpoints are on the circle. In the figure $\overline{BE}$ is the diameter.

Radius is a segment from the centre to one point of a circle. In the figure $\overline{AF}$ is a radius. The diameter is made up of two radii (plural for radius) namely $\overline{AE}$ and $\overline{AB}$. The radius is half of the diameter, while twice the radius is the diameter.

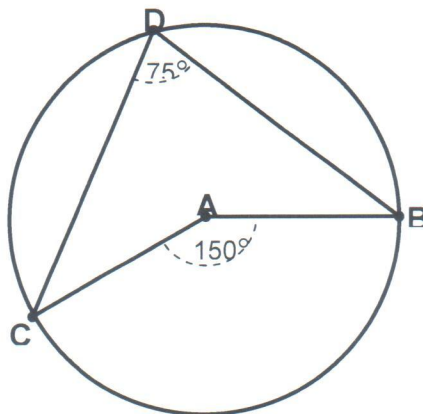
Chord is a segment whose endpoints are on the circle. The diameter is the longest chord of the circle. In the figure on the previous page, aside from the diameter, $\overline{CD}$ is a chord.

Arc is a part of the circle bounded by the chord. In the figure in the previous page, arc $\overline{CD}$ is the curve bounded by the chord $\overline{CD}$.

The following are theorems concerning circles.

Circle Theorem 1

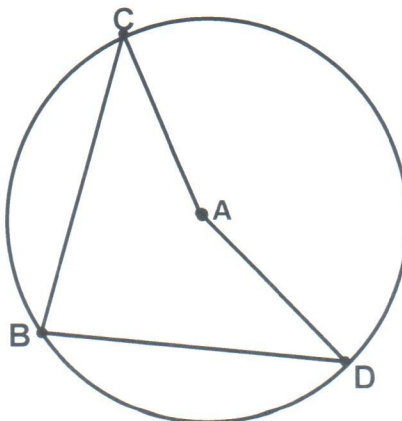
The angle at the centre is twice the angle at the circumference.



In the figure above, the angle at the centre is $\angle CAB$ which measure 150° . It is twice the measure of the angle at the circumference which is $\angle CDB$ measuring 75° .

Example 1

What is the measure of $\angle CDB$ if $\angle CAD$ measures 165° .

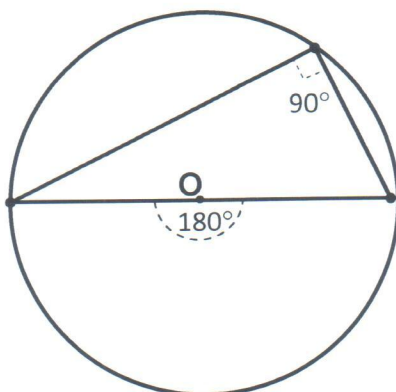


Solution:

Since the given is the angle at the center, which is twice the measure of the angle at the circumference, we divide its given measure by 2. That is $165 \div 2 = 82.5^\circ$.

Circle Theorem 2

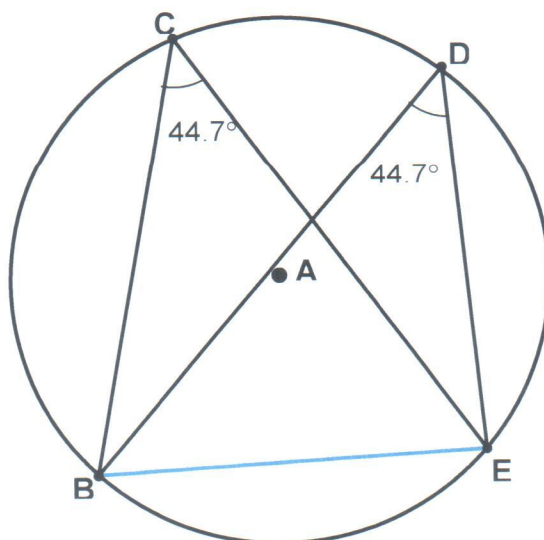
The angle in a semicircle is a right angle.



The figure illustrates that every angle in the circle which intercepts the diameter measures 90° .

Circle Theorem 3

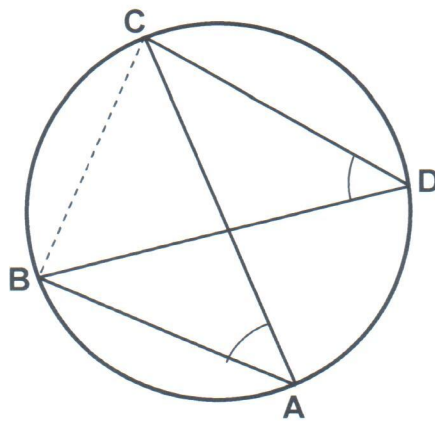
Angles in the same segment are equal.



The figure above shows both $\angle BCE$ and $\angle BDE$ intercept chord BE. Therefore, their measures are equal.

Example 2

What is the measure of $\angle \mathbf{BAC}$ if the measure of $\angle \mathbf{BDC}$ is 55° ?



Solution:

The measure of $\angle \mathbf{BAC}$ is also 55° because both angles $\mathbf{BAC}$ and $\mathbf{BDC}$ intercept chord $\mathbf{BC}$.

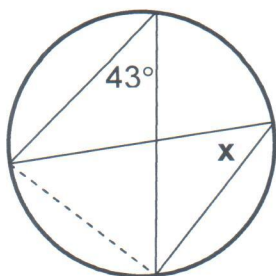
NOW DO PRACTICE EXERCISE 13



Practice Exercise 13

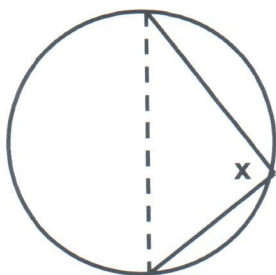
Find the measure of x in the following figures and state the theorem that supports your answer.

1)



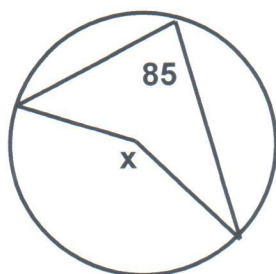
Answer: _____

2)



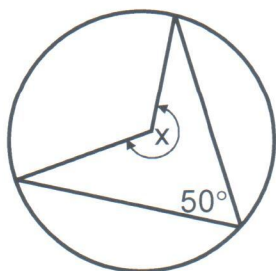
Answer: _____

3)



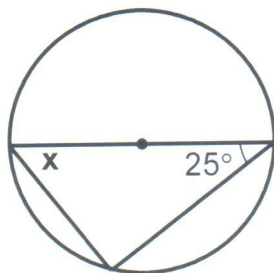
Answer: _____

4)



Answer: _____

5)



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 14: Tangent Theorems



You learnt something about circles in your previous Mathematics.



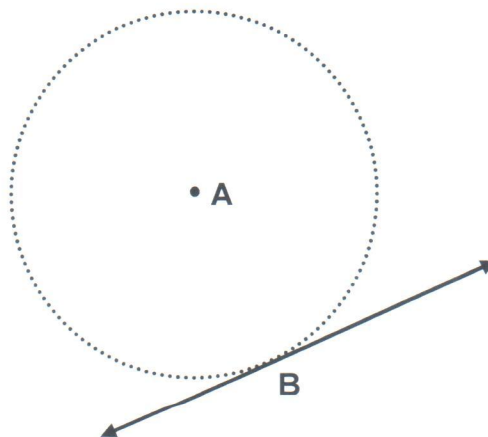
In this lesson, you will:

- define a tangent line in relation to a circle
- identify and explain the properties of tangent lines
- identify and explain tangent line theorems.

You learnt in your previous lesson that a **circle is a set of points equidistant to a fixed point called centre**. The basic parts of a circle were also discussed.

In this lesson we will discuss the other parts of a circle other than the basic parts discussed.

In the figure below, circle A is drawn. A line intersects the circle at point B. This line is called as the **tangent line**.



A **Tangent Line** is a line drawn outside the circle and it intersects the circle at one point. The point of intersection is called as the **point of tangency**.

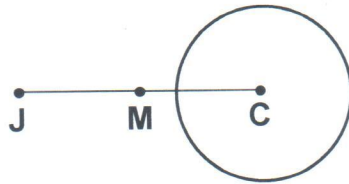
How to Construct Tangent Line to a circle using a compass and any straight edge?

There are several ways of drawing a line tangent to a circle. The following set of steps is just one way of doing so.

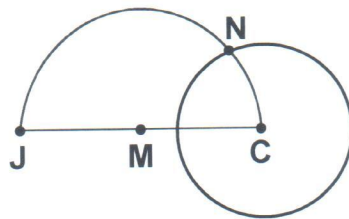
1. Given circle with centre C and a point J outside of the circle.



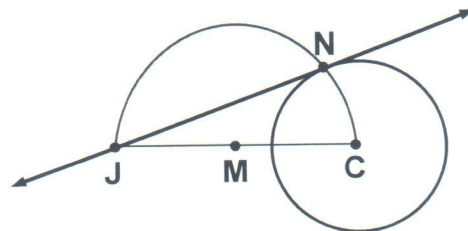
2. Connect point C and point J, find the midpoint M by bisecting the segment.



3. Using M as the centre, draw a semi-circle passing through points C and J. Name the intersection of the circle C and semi-circle as point N.



4. Construct a line through J and N.

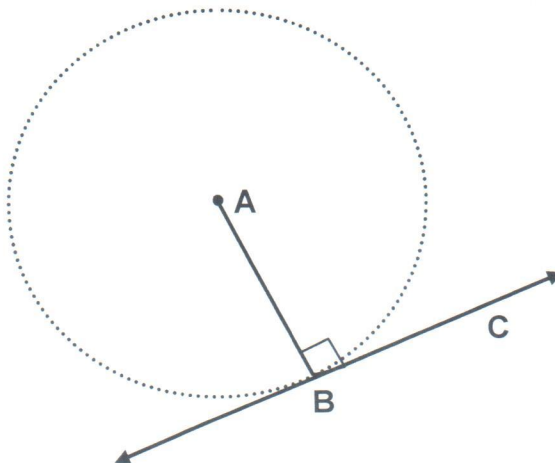


The $\overleftrightarrow{JN}$ is the desired tangent line.

The following are theorems concerning tangent lines.

Tangent Theorem 1

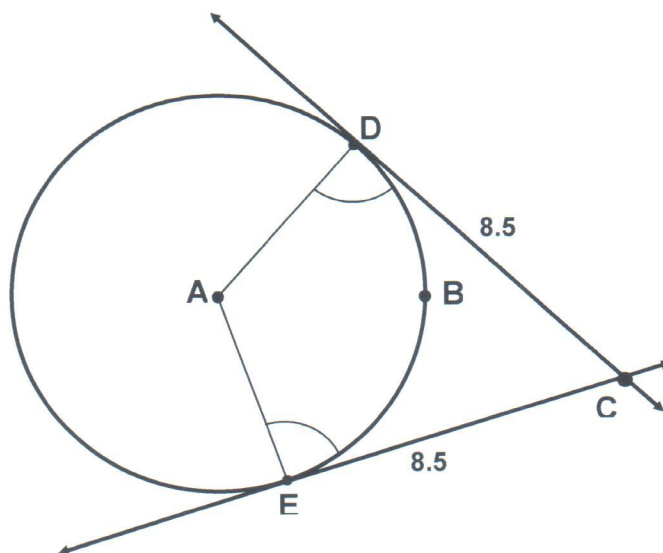
A radius and a tangent line forms a right angle at the point of tangency.



In the figure above, the radius **AB** intersects the tangent line **BC** at point **B**. Theorem 1 states that angle formed at point **B** is a right angle. Therefore angle **ABC** is a 90° angle.

Tangent Theorem 2

The lengths of the two tangents from a point to a circle are equal.

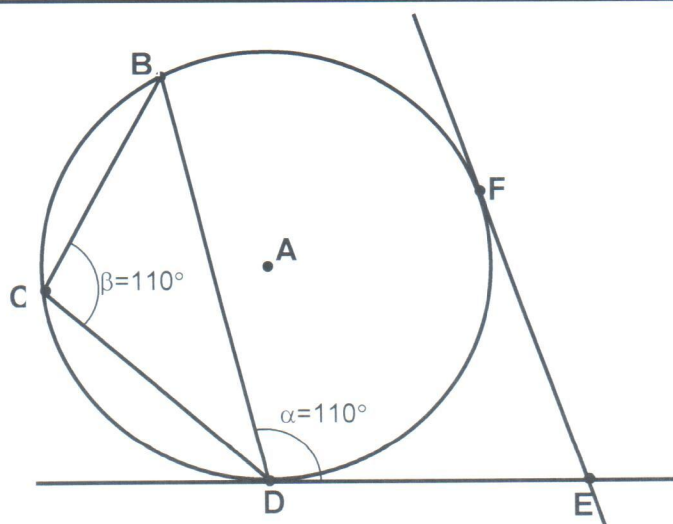


Notice that both $\overline{CD}$ and $\overline{CE}$ are congruent. The two lines pass through point **C**. The Tangent Theorem 2, states that, if both lines pass through a point outside the circle and both lines are tangent to the same circle, then there is an equal distance between the points outside the circle and their point of tangency.

This theorem is very useful because it saves time in proving the congruency of the two and it will also be very useful in future lessons dealing with practical applications of circles.

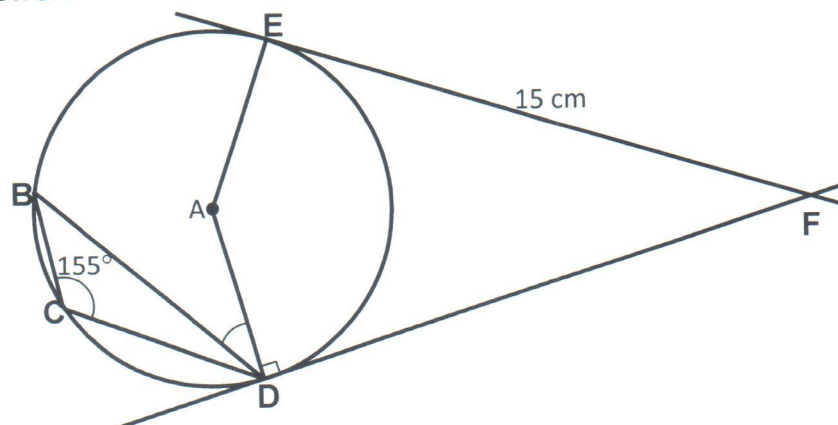
Tangent Theorem 3

Alternate segment theorem: The angle (α) between the tangent and the chord at the point of contact (D) is equal to the angle (β) in the alternate segment.



Example 1

Use the figure below to answer the following questions. State the Tangent Theorem that justifies the answer.



- Which angle in the figure is congruent with $\angle ADF$?
- What is the measure of $\angle ADB$?
- What is the measure of $\overline{DF}$?

Solution:

- $\angle AEF$ is congruent with $\angle ADF$.** Both angles are right angled. As the **Tangent Theorem 1** states that a radius and a tangent line forms a right angle at the point of tangency. Both $\overline{AE}$ and $\overline{AD}$ are radii of circle A. $\overline{AE}$ is tangent with $\overline{EF}$ at E, while the point of tangency of $\overline{AD}$ and $\overline{DF}$ is at D.
- The measure of $\angle ADB$ is derived by subtracting 90° from 155° . How did it happen? **Tangent Theorem 3** or known as the Alternate segment theorem states that $\angle BDF$ between the tangent and the chord at the point of contact D is equal to the angle $\angle BCD$ in the alternate segment. $\angle BDF$ measures 155° , while $\angle ADF$ measures 90° . That is $155^\circ - 90^\circ = 65^\circ$. **Therefore $\angle ADB$ measures 65° .**
- The measure of $\overline{DF}$ is equal to the measure of $\overline{EF}$. As **Tangent Theorem 2** states that the lengths of the two tangents $\overline{DF}$ and $\overline{EF}$ from a point F to a circle are equal. Since $\overline{EF}$ measures 15 cm, **therefore the measure of $\overline{DF}$ is 15 cm too.**

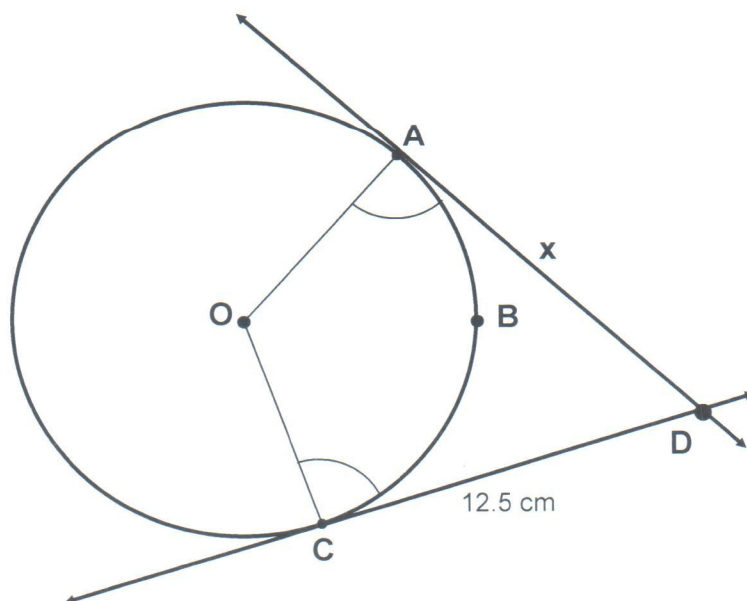
NOW DO PRACTICE EXERCISE 14

**Practice Exercise 14**

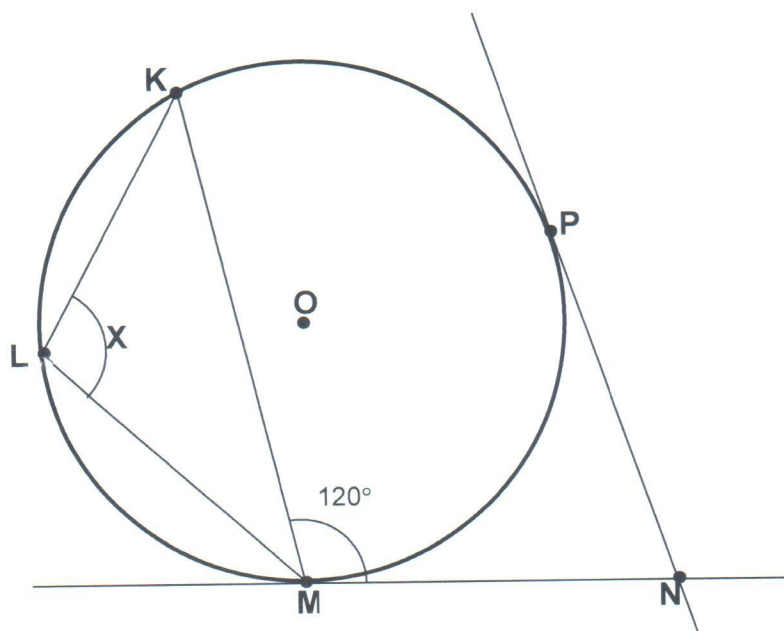
- 1) Given circle **X** and point **Y** outside the circle, draw a line with point **Y** tangent to the circle.

- 2) Find the measure of **x** in the following figures

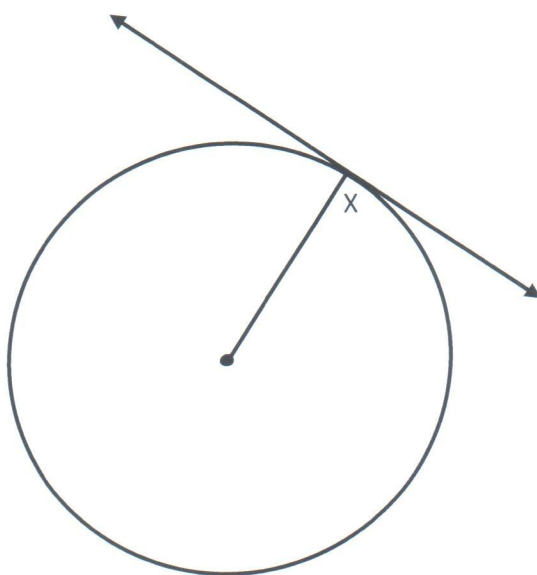
a)



b)



c)



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 15: General Problems Involving Circles



You learnt about circles and the different theorems concerning it.



In this lesson, you will:

- apply the theorems in solving general problems involving circles.

In this Lesson practical application of circles and lines related with it will be explored.

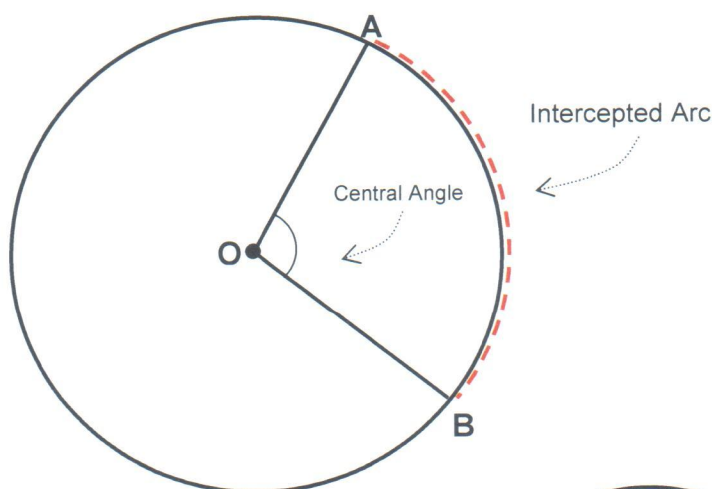
The following are formulas which will be very helpful in dealing with circle related problems.

The Central Angle Formula

The Central Angle is formed by two radii whose vertex is at the center of the circle. The central angle is equal to its intercepted arc.

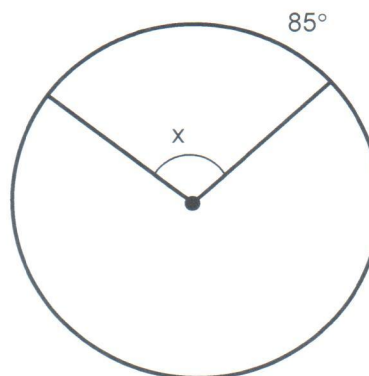
Using the figure below, the formula can be represented as

$$m\angle AOB = m \widehat{AB} \text{ (read as measure of arc } \widehat{AB}\text{)}$$



Example 1

Find the measure of the central angle in the figure.



Solution:

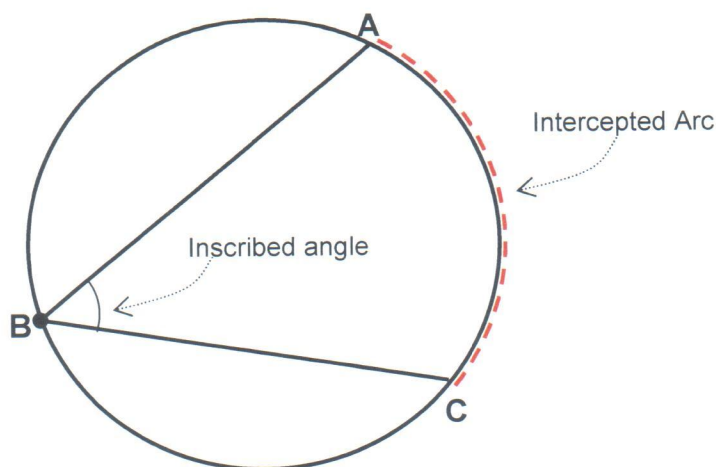
Without further computation, **the measure of the central angle is also 85°** . Since the formula states that The measure of the central angle is equal to the measure of the intercepted arc.

The Inscribed Formula

An inscribed angle is an angle with its vertex "on" the circle, formed by two intersecting chords. The measure of an inscribed angle is equal to one-half of the measure of the intercepted arc.

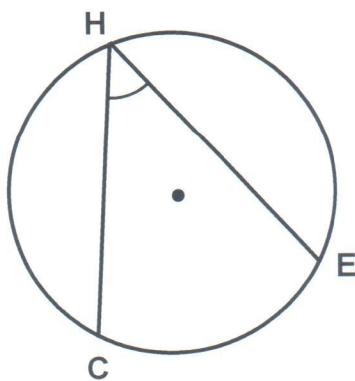
Using the figure below, the formula can be represented as

$$m\angle ABC = \frac{1}{2} m \widehat{AC}$$



Example 2

What is the measure of the inscribed angle CHE if its intercepted arc measures 120° ?



Solution:

Using the formula:

$$m\angle CHE = \frac{1}{2} m \widehat{CE}$$

We substitute the value of the given:

$$m\angle CHE = \frac{1}{2} (120^\circ)$$

Simplify:

$$m\angle CHE = 60^\circ$$

Therefore, the measure of $m\angle CHE$ is 60° .

Example 3

What is the measure of the intercepted arc if its inscribed angle measures 35° ?

Solution:

Although the figure is not drawn, knowing the formula concerning inscribed angle and its intercepted arc will help us solve the problem.

Using the formula:
$$m\angle ABC = \frac{1}{2} m\widehat{AC}$$

We substitute the value of the given:
$$35^\circ = \frac{1}{2} m\widehat{AC}$$

Cross multiply:
$$(35^\circ)(2) = m\widehat{AC}$$

Simplify:
$$70^\circ = m\widehat{AC}$$

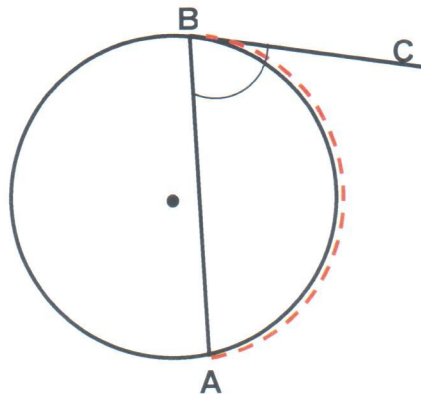
Therefore, the measure of the intercepted arc is 70°

The Tangent Chord Formula

A Tangent chord is an angle with its vertex "on" the circle formed by an intersecting chord and a tangent line. Its measure is equal to one-half of the measure of the intercepted arc.

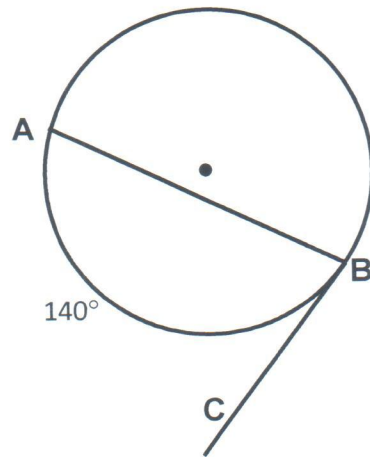
Using the figure below, the formula can be represented as

$$m\angle ABC = \frac{1}{2} m\widehat{AB}$$



Example 4

Find the measure of $\angle ABC$, given arc AB measuring 140° .



Solution:

Using the formula:

$$m\angle ABC = \frac{1}{2} m \widehat{AB}$$

We substitute the value of the given:

$$m\angle ABC = \frac{1}{2} (140^\circ)$$

Simplify:

$$m\angle ABC = 70^\circ$$

Therefore, the measure of m is 70° .

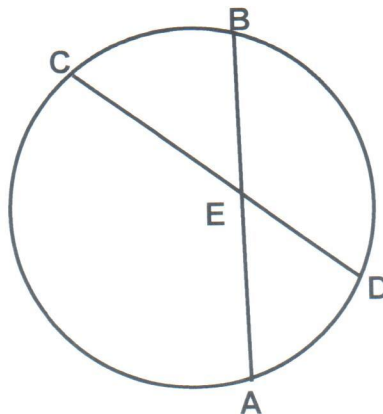
The Intersecting Chords Formula

An angle formed by two intersecting chords whose vertex is inside the circle measures one-half of the sum of the measures of the two intercepted arcs.

Using the figure below, the formula can be represented as

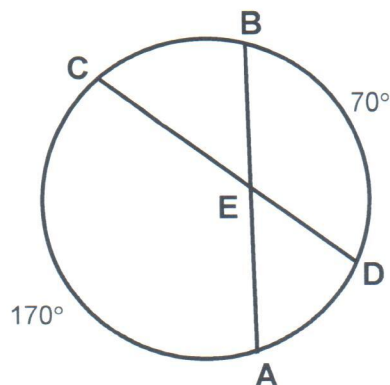
$$m\angle BED = \frac{1}{2} (m \widehat{BD} + m \widehat{AC})$$

Note: $\angle BED$ and $\angle CEA$ are congruent because they are vertical angles. Therefore the same formula applies in solving for their measures.



Example 5

Find the measure of $\angle BED$ in the figure.



Solution:

Using the formula:

$$m\angle BED = \frac{1}{2}(m\widehat{BD} + m\widehat{AC})$$

We substitute the value of the given: $m\angle BED = \frac{1}{2}(70^\circ + 170^\circ)$

Simplify:

$$m\angle BED = \frac{1}{2}(240^\circ)$$

$$m\angle BED = 120^\circ$$

Therefore, the measure of $m\angle BED$ is 120° .

The Angle Formed Outside the Circle Formula

An angle formed by two intersecting chords (secant lines) or tangent lines whose vertex is outside the circle measures one-half of the difference of the measures of the two intercepted arcs.

The general formula applies

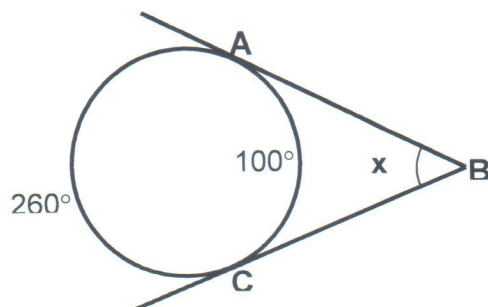
$$\text{Angle Formed Outside} = \frac{1}{2} \text{Difference of Intercepted Arcs}$$

Note: When subtracting, start with the larger arc.

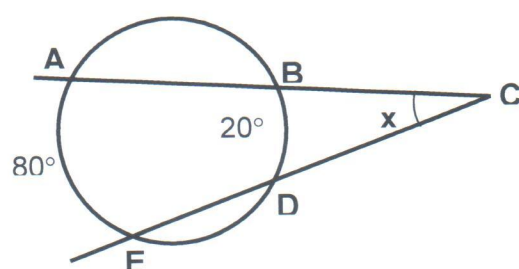
Example 6

Find the measures of the unknown represented by x in the following figures:

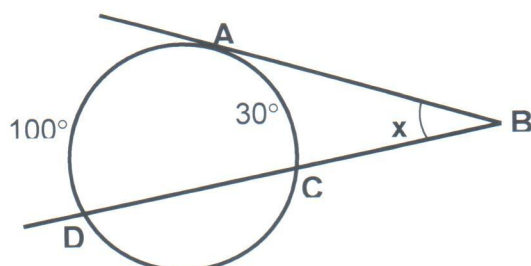
a)



b)



c)



Solution:

a) Using the formula:
$$m\angle ABC = \frac{1}{2} (\text{major arc } \widehat{AC} - \text{minor arc } \widehat{AC})$$

We substitute the value of the given:
$$m\angle ABC = \frac{1}{2} (260^\circ - 100^\circ)$$

Simplify:
$$m\angle ABC = \frac{1}{2} (160^\circ)$$

$$m\angle ABC = 80^\circ$$

Therefore, the measure of $\angle ABC$ is 80° .

b) Using the formula: $m\angle ACE = \frac{1}{2} (m \widehat{AE} - m \widehat{BD})$

We substitute the value of the given: $m\angle ACE = \frac{1}{2} (80^\circ - 20^\circ)$

Simplify: $m\angle ACE = \frac{1}{2} (60^\circ)$

$$m\angle ACE = 30^\circ$$

Therefore, the measure of $\angle ACE$ is 30° .

c) Using the formula: $m\angle ABD = \frac{1}{2} (m \widehat{AD} - m \widehat{AC})$

We substitute the value of the given: $m\angle ABD = \frac{1}{2} (100^\circ - 30^\circ)$

Simplify: $m\angle ABD = \frac{1}{2} (70^\circ)$

$$m\angle ABD = 35^\circ$$

Therefore, the measure of $\angle ABD$ is 35° .

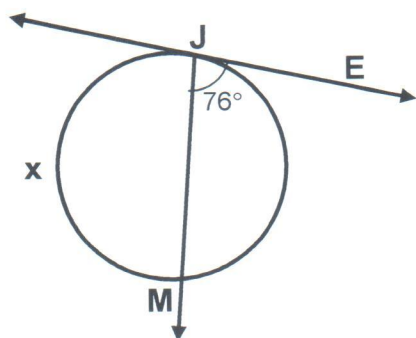
NOW DO PRACTICE EXERCISE 15



Practice Exercise 15

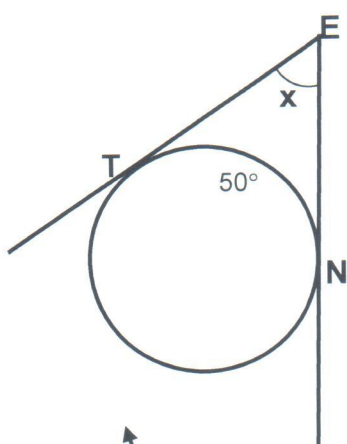
Solve for the unknown measure of the angle or arc in the figure represented by x .

1.



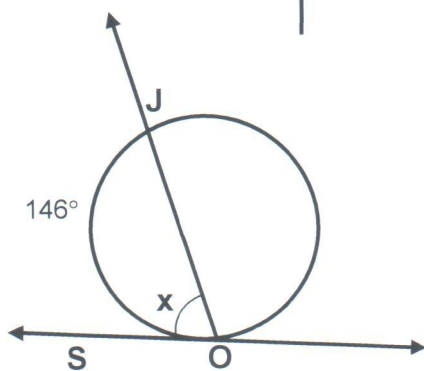
Answer: _____

2.



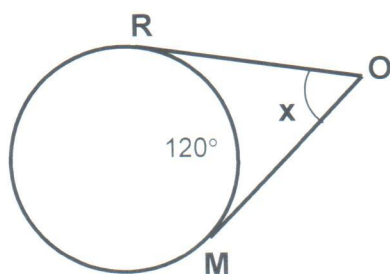
Answer: _____

3.



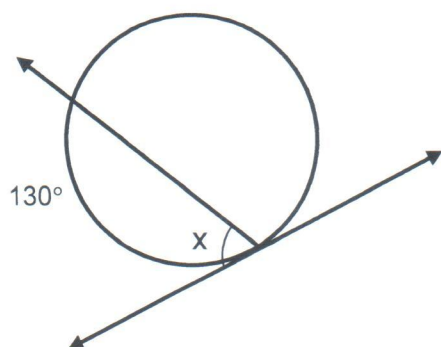
Answer: _____

4.



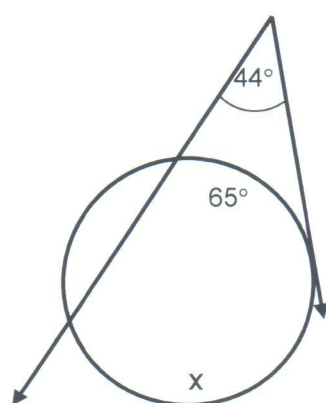
Answer: _____

5.



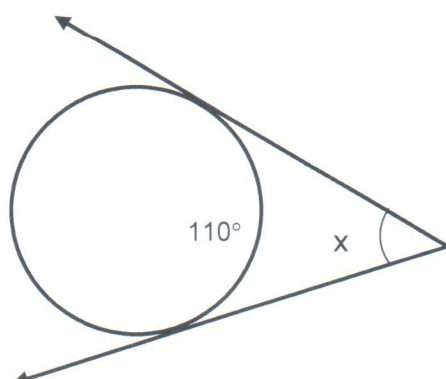
Answer: _____

6.



Answer: _____

7.



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 16: Circle problems Associated with Pythagoras' Theorem



You learnt about some applications of learning theorems concerning circles in the previous lesson.



In this lesson, you will:

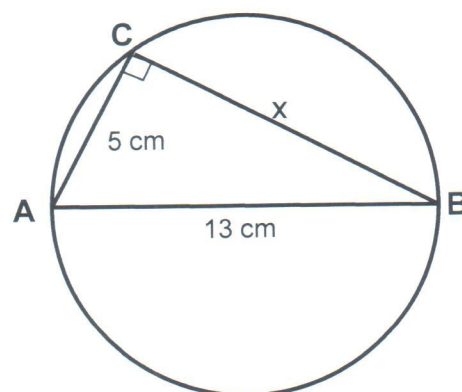
- apply Pythagoras' theorem to solve circle problems.

There are certain properties of circles which may involve right angles. In these situations we can use the Pythagoras' Theorem.

The following are some circle problems which may require the use of the Pythagoras' Theorem.

Example 1

A circle has diameter $[AB]$ of length 13 cm. C is a point on the circle such that AC is 5 cm. Find the length BC .



Solution:

Since the angle formed in a semicircle is a right angle, the Pythagoras' Theorem could be used.

Since the hypotenuse is the diameter we say

$$13^2 = 5^2 + x^2$$

We transpose 5^2 to isolate x^2 on one side of the equation

$$13^2 - 5^2 = x^2$$

Simplify: $169 - 25 = x^2$

$$144 = x^2$$

Get the square root of both sides of the equation

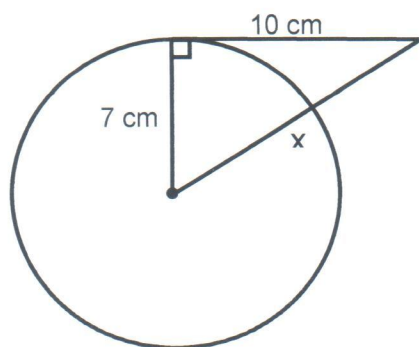
$$\sqrt{144} = x$$

$$12 = x$$

Therefore, the measure of BC is 12 cm.

Example 2

A tangent of length 10 cm is drawn to a circle with radius 7 cm. How far is the center of the circle from the end point of the tangent?



Solution:

Since the angle formed by a radius and a tangent line is a right angle, the Pythagoras' Theorem could be used.

Since the hypotenuse (x) is the unknown we say

$$x^2 = 7^2 + 10^2$$

Simplify: $x^2 = 49 + 100$

$$x^2 = 149$$

Get the square root of both sides of the equation

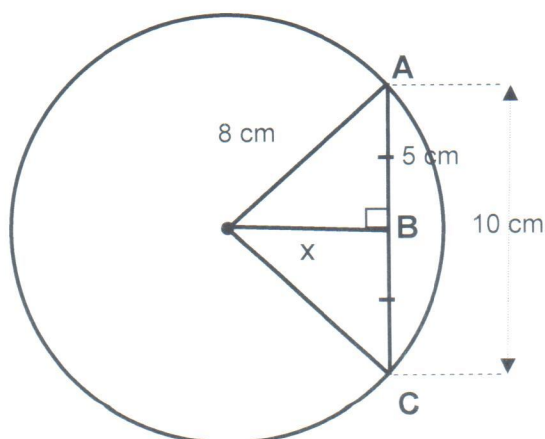
$$x = \sqrt{149}$$

$$x = 12.2$$

Therefore, the distance between the centre of the circle and the endpoint of the tangent line is 12.2 cm.

Example 3

A circle has a chord of length 10 cm. If the radius of the circle is 8 cm, find the shortest distance from the center of the circle to the chord.



Solution:

Since the triangle formed using the radii is an isosceles triangle, and the shortest distance from the centre to the chord forms a right angle the Pythagoras' Theorem could be used.

Since the hypotenuse is 8 cm and adjacent side is 5 cm we say

$$8^2 = 5^2 + x^2$$

We transpose 5^2 to isolate x^2 on one side of the equation

$$8^2 - 5^2 = x^2$$

Simplify:

$$64 - 25 = x^2$$

$$39 = x^2$$

Get the square root of both sides of the equation

$$\sqrt{39} = x$$

$$6.24 = x$$

Therefore, the shortest distance between the centre and the chord is 6.24 cm.

NOW DO PRACTICE EXERCISE 16



Practice Exercise 16

Answer the following word problems. You may sketch if you need to.

1. A chord of a circle has length 3 cm. If the circle has radius 4 cm, find the shortest distance from the center of the circle to the chord.
2. A circle has radius 3 cm. A tangent is drawn to the circle from point P which is 9 cm from O, the circle's center. How long is the tangent?
3. A rectangle with side lengths 11 cm and 6 cm is inscribed in a circle. Find the diameter of the circle.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

TOPIC 3: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- A **quadrilateral** is a four-sided polygon. It has four sides (edges), four vertices (corners) and its interior angles add up to 360 degrees.
- **Parallelogram** is a quadrilateral with two pairs of parallel sides and opposite sides congruent.
- **Rectangle** is a parallelogram with 4 right angles.
- **Square** is both a rhombus and a rectangle. It is a rhombus because it has 4 equal sides. It is a rectangle because it has 4 right angles.
- **Trapezoid** (called a trapezium in the UK) has a pair of opposite sides parallel. It is called an **Isosceles trapezoid** if the sides that aren't parallel are equal in length and both angles coming from a parallel side are equal
- **General Quadrilaterals** are any quadrilaterals with no parallel sides. The most common of these is the **KITE** with special properties.
- The **Interior Angle Sum** of a polygon can be represented by the notation:
 $S = (n-2) 180^\circ$ where: **S** is the interior angle sum, and **n** is the number of sides of a polygon.
- A **circle** is a set of points equidistant to a fixed point called centre. The centre of a circle is named using a capital letter.
- **Circle Theorems 1, 2, and 3**
 1. The angle at the centre is twice the angle at the circumference.
 2. The angle in a semicircle is a right angle.
 3. Angles in the same segment are equal.
- A **Tangent Line** is a line drawn outside the circle and it intersects the circle at one point. The point of intersection is called as the point of tangency.
- **Tangent Theorem 1, 2, and 3:**
 1. A radius and a tangent line forms a right angle at the point of tangency.
 2. The lengths of the two tangents from a point to a circle are equal.
 3. (Alternate segment theorem): The angle (α) between the tangent and the chord at the point of contact (D) is equal to the angle (β) in the alternate segment.
- The **Central Angle** is formed by two radii whose vertex is at the centre of the circle. The central angle is equal to its intercepted arc.
- An **inscribed angle** is an angle with its vertex "on" the circle, formed by two intersecting chords. The measure of an inscribed angle is equal to one-half of the measure of the intercepted arc.
- An angle with its vertex "on" the circle, formed by intersecting chord and a tangent line. Its measure is equal to one-half of the measure of the intercepted arc.
- An angle formed by two intersecting chords whose vertex is inside the circle measures one-half of the sum of the measures of the two intercepted arcs.
- An angle formed by two intersecting chords (secant lines) or tangent lines whose vertex is outside the circle measures one-half of the difference of the measures of the two intercepted arcs.

REVISE LESSONS 10 TO 16. THEN DO TOPIC TEST 3 IN ASSIGNMENT BOOK 5.

ANSWERS TO PRACTICE EXERCISES 10-16

Practice Exercise 10

1.
 - a) TRUE
 - b) FALSE
 - c) TRUE
 - d) TRUE
 - e) FALSE
 - f) FALSE
 - g) TRUE
 - h) TRUE
 - i) TRUE
 - j) TRUE
 2.
 - a) Rhombus
 - b) Rectangle
 - c) Square
 3.
 - a) $x = 115^\circ$ $y = 65^\circ$
 - b) $x = 75^\circ$ $y = 110^\circ$
-

Practice Exercise 11

1.
 - a) 1260°
 - b) 1080°
 - c) 1620°
 - d) 2340°
 2.
 - a) 120°
 - b) 135°
 - c) 152.31°
 - d) 157.5°
-

Practice Exercise 12

1.
 - a. Addition property of equality
 - b. Division property of Equality
 - c. Substitution

2. a.

Statement	Reason
1. J is the midpoint of $\overline{MK}$	1. Given
2. K is the midpoint of $\overline{JL}$	2. Given
3. $\overline{MJ} \cong \overline{JK}$	3. Definition of midpoint
4. $\overline{JK} \cong \overline{KL}$	4. Definition of midpoint
5. $\overline{MJ} \cong \overline{KL}$	5. Substitution

b.

Statement	Reason
1. $\triangle GES \cong \triangle STG$	1. Given
2. $\overline{GT} \cong \overline{SE}$	2. CPCTC
3. $\angle ESA \cong \angle TGA$	3. Given
4. $\angle A \cong \angle A$	4. Identity
5. $\triangle ASE \cong \triangle AGT$	5. SAA Congruency
6. $\overline{SA} \cong \overline{GA}$	6. CPCTC

3.

Statement	Reason
1. $\angle BAC \cong \angle DAE$, $\overline{AE} \cong \overline{AC}$ A is the midpoint of $\overline{BD}$	1. Given
2. $m\angle BAC = m\angle DAE$	2. Congruent angles are of equal measure
3. $m\angle 2 = m\angle 2$	3. Identity
4. $m\angle DAE = m\angle 2 + m\angle 3$, $m\angle BAC = m\angle 1 + m\angle 2$	4. Angle Addition Postulate(A whole is equal to its parts)
5. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	5. Substitution
6. $m\angle 1 = m\angle 3$	6. Subtraction Property
7. $\angle 1 \cong \angle 3$	7. Congruent angles are equal
8. $\overline{AB} = \overline{AD}$	8. Definition of midpoint
9. $\triangle BEA \cong \triangle DCA$	9. SAS Congruency

TOPIC 4

INTRODUCTION TO TRIGONOMETRIC RATIOS

Lesson 17:	What is Trigonometry?
Lesson 18:	The Sine Ratio
Lesson 19:	The Cosine Ratio
Lesson 20:	The Tangent Ratio
Lesson 21:	Mixed Problems

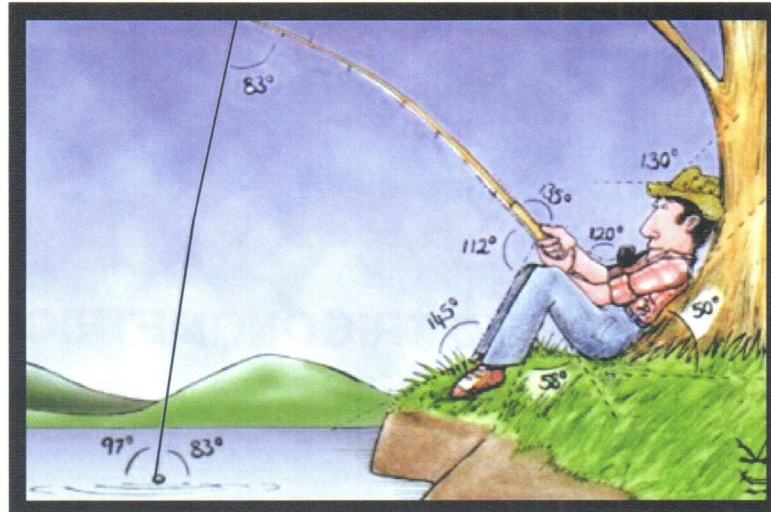
TOPIC 4: TRIGONOMETRY

Introduction



What is trigonometry?

The word **Trigonometry** was derived from the Greek words **trigon** meaning "triangle" (from tri- "three" + gonia "angle") and **metron** which means "a measure".



Deriving its meaning from the etymology, it simply means "triangle measure". These basic words will define how trigonometry really works. It is founded on the knowledge of measuring triangles specifically right triangles and oblique triangles (non-right triangle).

Land survey makes use of the process of triangulation, in which a chosen network of triangles is measured. In pure triangulation, a base line to one triangle is measured, and the rest of the survey involves measuring angles only; in mixed triangulation, certain sides and angles are measured; in chain triangulation, only sides are measured.

Trigonometric concepts are used to minimize the amount of measuring involved. These concepts depend on the concepts of enlargement and similarity. Equiangular triangles have the same shape, but only in the special case of congruency do they have the same size. Any set of similar triangles has the invariant property of proportionality; that is, ratios of pairs of corresponding sides are in the same proportion. In the language of transformation geometry, for similar triangles, one triangle is an enlargement of another; or any triangle can be transformed into another by applying the same scale factor to each part of the triangle.

This Topic encompasses the fundamentals of trigonometry. It involves lessons on naming and measuring angles to the application of the fundamental trigonometric ratios. These knowledge will serve as the foundation for higher trigonometric lessons.

Lesson 17: What is Trigonometry?



You learnt in the previous lessons practical applications of circles and triangles.



In this lesson, you will:

- define what Trigonometry is
- explain the purpose of using trigonometry
- describe the ratios of the lengths of the sides of right-angled triangles.

What is trigonometry?

Trigonometry is a branch of mathematics that developed from simple measurement of geometric quantities, geometry, and surveying.

It makes use of concepts from algebra and analysis. Basically it involved the mathematics of practical problems, such as construction and land measurement; it has since been extended to the geometry of three-dimensional spaces.

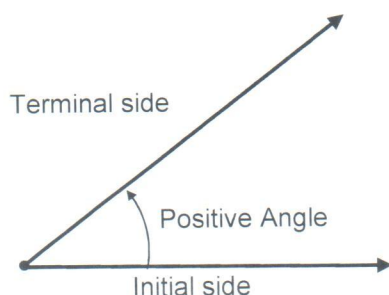
In this lesson, Trigonometry is only limited to its use in solving problems involving the angles and sides of right-angled triangles.



Angle Measures in Trigonometry

Since Trigonometry deals with measurements of angles and triangles, it is important to note that the way angles are measured in geometry are different with the way they are measured in Trigonometry.

The figure below shows a trigonometric angle.

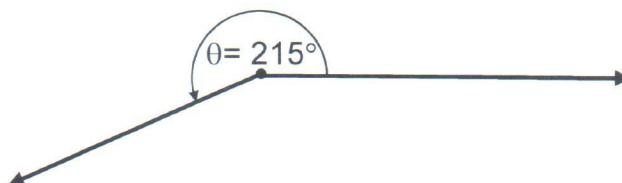
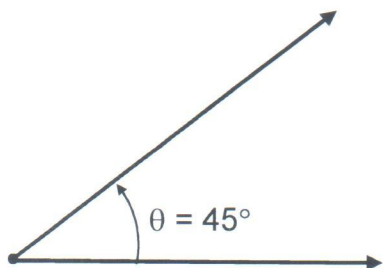


The **initial side** is the leg of an angle which always lies horizontally. The initial side does not move. It always stick to its position.

The **terminal side** is the leg of an angle which determines the rotation of the angle. This side can move in either clockwise or anti-clockwise direction.

Angles in trigonometry can either be positive or negative depending on the direction of rotation of the terminal side. Unlike in geometry, angle measure range from 0° - 180° . In trigonometry the measure of an angle could be any real number both positive and negative.

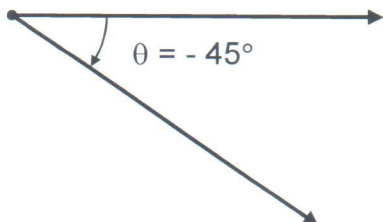
Observe the following angles:



Did you notice the rotation of the angle?

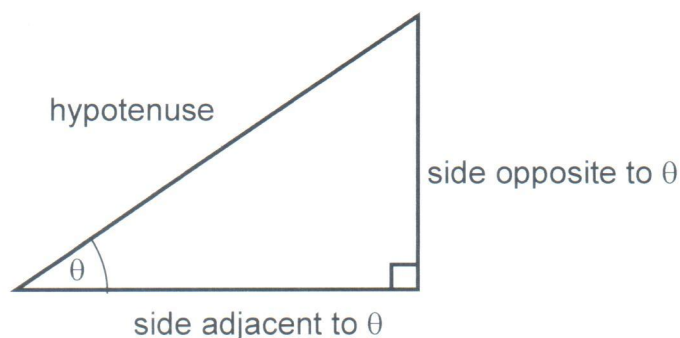
For positive angles, the rotation is anti-clockwise or counter-clockwise.

Negative angles have opposite rotation which is clockwise as shown in the following figure:



The Trigonometric Ratios

Given a right triangle, the basic trigonometric ratios could be determined.



In a right angled triangle, the side opposite to the right angle is called the hypotenuse. If we choose one of the other angles and label it θ (theta) the other sides are often called opposite (the side opposite to θ) and adjacent (the side next to θ).

For every right triangle labelled as the one above the ratios of corresponding sides have the same values.

The ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ has the same value for each triangle. This ratio is given a special name, the cosine of θ or **$\cos \theta$** .

The ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ has the same value for each triangle. This ratio is the sine of θ or **$\sin \theta$** .

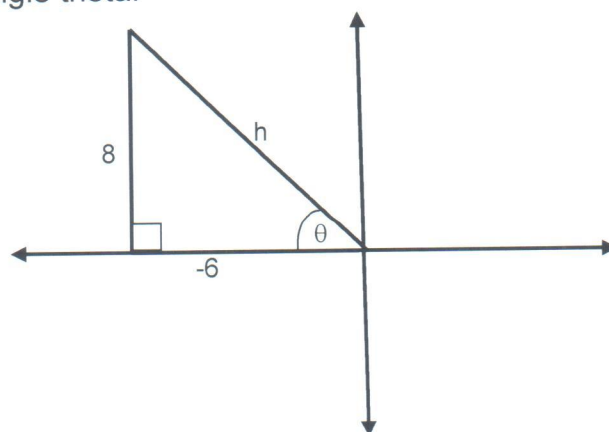
The ratio $\frac{\text{opposite}}{\text{adjacent}}$ takes the same value for each triangle. This ratio is called the tangent of θ or **$\tan \theta$** .

These ratios are called **primary trigonometric ratios** and can be remembered by the acronym **SOH – CAH – TOA**,

where: **SOH** stands for Sine = opposite / hypotenuse
CAH stands for Cosine = adjacent / hypotenuse
TOA stands for Tangent = opposite / adjacent

Example 1

Find the exact values of the primary trigonometric ratios given a point P(-6, 8) on the terminal side of angle theta.



Solution:

Solve for the hypotenuse first before we could identify the trigonometric ratios.

$$\begin{aligned}\text{hypotenuse} &= \sqrt{8^2 + (-6)^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10\end{aligned}$$

With respect to the angle θ , its opposite side measures 8 units ; adjacent side is -6 units and its hypotenuse measure 10 units. With these data, we can now come up with the primary trigonometric functions.

The primary trigonometric ratios are:

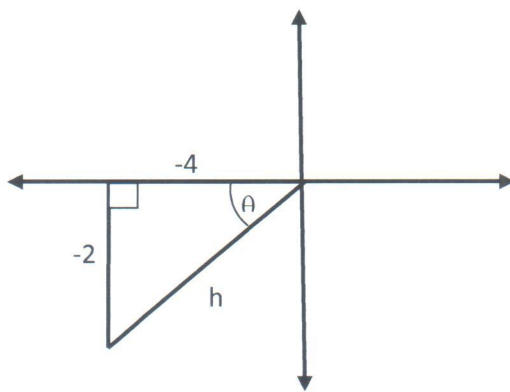
$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{-6}{10} = -\frac{3}{5}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent}} = \frac{8}{-6} = -\frac{4}{3}$$

Example 2

Find the exact values of the primary trigonometric ratios given a point P(-4, -2) on the terminal side of angle theta.



Solution:

Solve for the hypotenuse first before we could identify the trigonometric ratios.

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{(-4)^2 + (-2)^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

The primary trigonometric ratios are:

$$\sin A = \frac{-2}{2\sqrt{5}} = \frac{-1}{\sqrt{5}}$$

$$\cos A = \frac{-4}{2\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

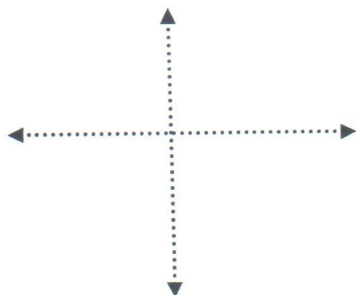
$$\tan A = \frac{-2}{-4} = \frac{1}{2}$$

NOW DO PRACTICE EXERCISE 17

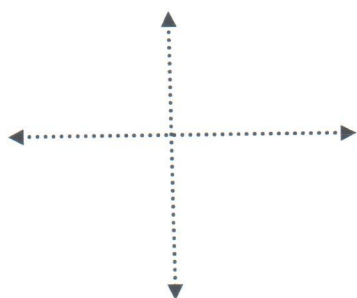
**Practice Exercise 17**

1. Draw the following angles

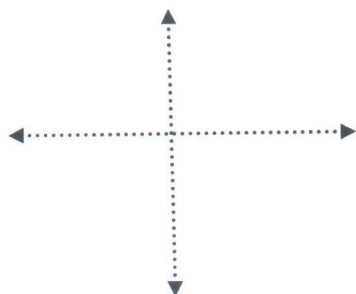
a) 80°



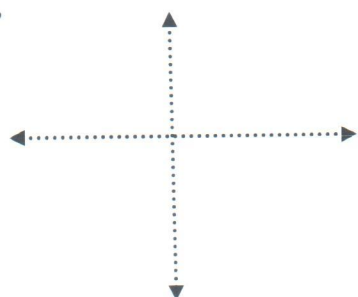
b) -80°



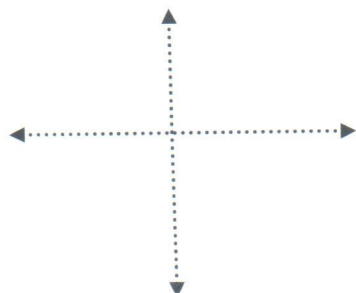
c) 45°



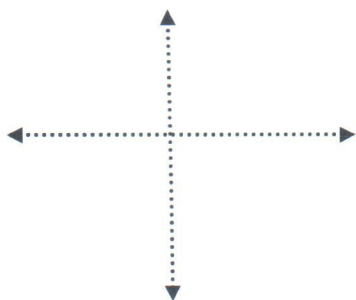
d) -155°



e) 270°



f) -185°



2. Find the primary trigonometric ratios given the following points on the terminal side of angle A.

a) $P(-3, -4)$

b) $P(2, -5)$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 18: The Sine Ratio



You have learned the primary trigonometric ratios in the previous lesson.



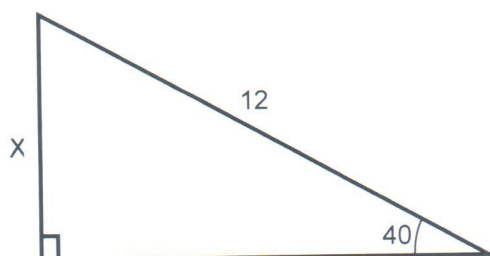
In this lesson, you will:

- define and explain the sine ratio
- use trigonometric table to find the sine ratio for a given angle
- find an angle and the length of a side of a right-angled triangle given its sine ratio.

We have discussed in Lesson 17 the primary trigonometric ratios. In this lesson we will have a thorough study of the sine ratio or the ratio formed out of the opposite side and hypotenuse of a right triangle.

To use this ratio we must have at least given values of any two of the angle, hypotenuse and opposite side of the right triangle.

Example 1



Solution:

Form the sine ratio: $\sin 40^\circ = \frac{x}{12}$

Cross multiply: $(\sin 40^\circ)(12) = x$

To get $\sin 40^\circ$, you can use a scientific calculator or you can use the trigonometric table for sine function found on the next page.

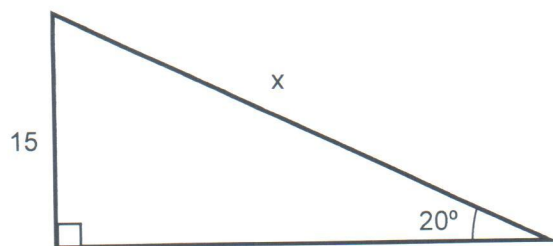
Simplify $(0.643)(12) = x$
 $7.716 = x$

Therefore, the value of the opposite side is 7.72.

Trigonometry Table (Sine Ratio)					
Radian	Degree	Sine	Radian	Degree	Sine
0.000	0	0.000	0.803	46	0.719
0.017	1	0.017	0.820	47	0.731
0.035	2	0.035	0.838	48	0.743
0.052	3	0.052	0.855	49	0.755
0.070	4	0.070	0.873	50	0.766
0.087	5	0.087	0.890	51	0.777
0.105	6	0.105	0.908	52	0.788
0.122	7	0.122	0.925	53	0.799
0.140	8	0.139	0.942	54	0.809
0.157	9	0.156	0.960	55	0.819
0.175	10	0.174	0.977	56	0.829
0.192	11	0.191	0.995	57	0.839
0.209	12	0.208	1.012	58	0.848
0.227	13	0.225	1.030	59	0.857
0.244	14	0.242	1.047	60	0.866
0.262	15	0.259	1.065	61	0.875
0.279	16	0.276	1.082	62	0.883
0.297	17	0.292	1.100	63	0.891
0.314	18	0.309	1.117	64	0.899
0.332	19	0.326	1.134	65	0.906
0.349	20	0.342	1.152	66	0.914
0.367	21	0.358	1.169	67	0.921
0.384	22	0.375	1.187	68	0.927
0.401	23	0.391	1.204	69	0.934
0.419	24	0.407	1.222	70	0.940
0.436	25	0.423	1.239	71	0.946
0.454	26	0.438	1.257	72	0.951
0.471	27	0.454	1.274	73	0.956
0.489	28	0.469	1.292	74	0.961
0.506	29	0.485	1.309	75	0.966
0.524	30	0.500	1.326	76	0.970
0.541	31	0.515	1.344	77	0.974
0.559	32	0.530	1.361	78	0.978
0.576	33	0.545	1.379	79	0.982
0.593	34	0.559	1.396	80	0.985
0.611	35	0.574	1.414	81	0.988
0.628	36	0.588	1.431	82	0.990
0.646	37	0.602	1.449	83	0.993
0.663	38	0.616	1.466	84	0.995
0.681	39	0.629	1.484	85	0.996
0.698	40	0.643	1.501	86	0.998
0.716	41	0.656	1.518	87	0.999
0.733	42	0.669	1.536	88	0.999
0.750	43	0.682	1.553	89	1.000
0.768	44	0.695	1.571	90	1.000
0.785	45	0.707			

Example 2

Find the value of x (measure of the hypotenuse) in the figure below.



Solution:

Form the sine ratio: $\sin 20^\circ = \frac{15}{x}$

Cross multiply: $x = \frac{15}{\sin 20^\circ}$

To get $\sin 20^\circ$, you can use a scientific calculator or you can use the trigonometric table for sine function found on the previous page.

Simplify $x = \frac{15}{0.342}$
 $x = 43.859$

Therefore, the value of the hypotenuse is 43.86.

NOW DO PRACTICE EXERCISE 18

**Practice Exercise 18**

1) Using the trigonometric table for Sine Ratio, Find the values of the following:

a) $\sin 45^\circ$ _____

b) $\sin 15^\circ$ _____

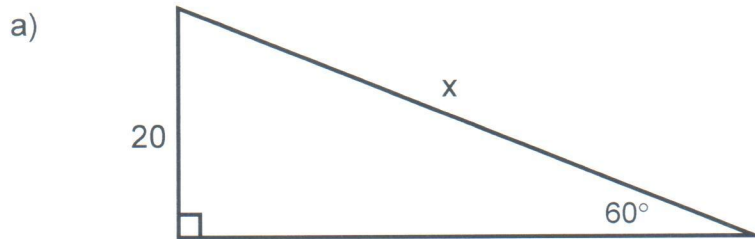
c) $\sin 30^\circ$ _____

d) $\sin 70^\circ$ _____

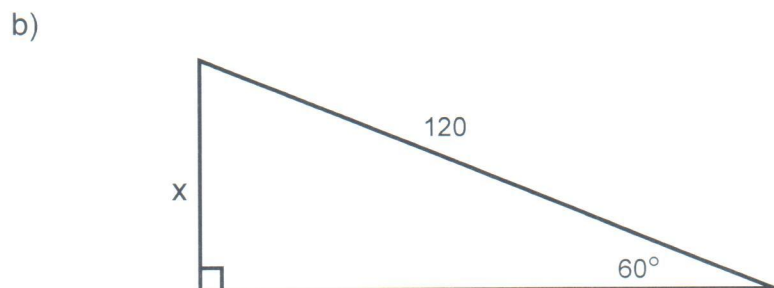
e) $\sin 85^\circ$ _____

f) $\sin 90^\circ$ _____

2) Solve for x in the following figures



Answer: _____



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 19: The Cosine Ratio



You have learned the primary trigonometric ratios in the previous lesson.



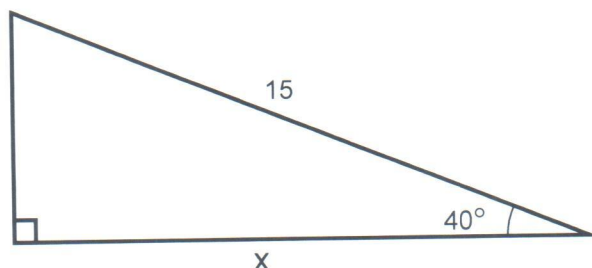
In this lesson, you will:

- define and explain the cosine ratio
- use trigonometric table to find the cosine ratio for a given angle
- find an angle and the length of a side of a right-angled triangle given its cosine ratio.

We have discussed in Lesson 17 the primary trigonometric ratios. In this lesson we will have a thorough study of the cosine ratio or the ratio formed out of the adjacent side and hypotenuse of a right triangle.

To use this ratio we must have at least given values of any two of the angle, hypotenuse and adjacent side of the right triangle.

Example 1



Solution:

Form the cosine ratio: $\cos 40^\circ = \frac{x}{15}$

Cross multiply: $(\cos 40^\circ)(15) = x$

To get $\cos 40^\circ$, you can use a scientific calculator or you can use the trigonometric table for cosine function found on the next page.

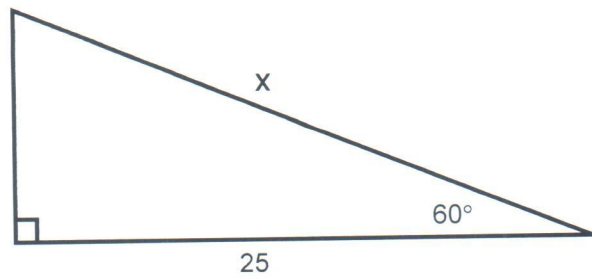
Simplify $(0.766)(15) = x$
 $11.49 = x$

Therefore, the value of the adjacent side is 11.49.

Trigonometry Table Cosine Ratio					
Radian	Degree	Cosine	Radian	Degree	Cosine
0.000	0	1.000	0.803	46	0.695
0.017	1	1.000	0.820	47	0.682
0.035	2	0.999	0.838	48	0.669
0.052	3	0.999	0.855	49	0.656
0.070	4	0.998	0.873	50	0.643
0.087	5	0.996	0.890	51	0.629
0.105	6	0.995	0.908	52	0.616
0.122	7	0.993	0.925	53	0.602
0.140	8	0.990	0.942	54	0.588
0.157	9	0.988	0.960	55	0.574
0.175	10	0.985	0.977	56	0.559
0.192	11	0.982	0.995	57	0.545
0.209	12	0.978	1.012	58	0.530
0.227	13	0.974	1.030	59	0.515
0.244	14	0.970	1.047	60	0.500
0.262	15	0.966	1.065	61	0.485
0.279	16	0.961	1.082	62	0.469
0.297	17	0.956	1.100	63	0.454
0.314	18	0.951	1.117	64	0.438
0.332	19	0.946	1.134	65	0.423
0.349	20	0.940	1.152	66	0.407
0.367	21	0.934	1.169	67	0.391
0.384	22	0.927	1.187	68	0.375
0.401	23	0.921	1.204	69	0.358
0.419	24	0.914	1.222	70	0.342
0.436	25	0.906	1.239	71	0.326
0.454	26	0.899	1.257	72	0.309
0.471	27	0.891	1.274	73	0.292
0.489	28	0.883	1.292	74	0.276
0.506	29	0.875	1.309	75	0.259
0.524	30	0.866	1.326	76	0.242
0.541	31	0.857	1.344	77	0.225
0.559	32	0.848	1.361	78	0.208
0.576	33	0.839	1.379	79	0.191
0.593	34	0.829	1.396	80	0.174
0.611	35	0.819	1.414	81	0.156
0.628	36	0.809	1.431	82	0.139
0.646	37	0.799	1.449	83	0.122
0.663	38	0.788	1.466	84	0.105
0.681	39	0.777	1.484	85	0.087
0.698	40	0.766	1.501	86	0.070
0.716	41	0.755	1.518	87	0.052
0.733	42	0.743	1.536	88	0.035
0.750	43	0.731	1.553	89	0.017
0.768	44	0.719	1.571	90	0.000
0.785	45	0.707			

Example 2

Find the value of x in the figure below.



Solution

Form the cosine ratio: $\cos 60^\circ = \frac{25}{x}$

Cross multiply: $x = \frac{25}{\cos 60^\circ}$

To get $\cos 60^\circ$, you can use a scientific calculator or you can use the trigonometric table for sine function found on the previous page.

Simplify $x = \frac{25}{0.5}$
 $x = 50$

Therefore, the value of x or the measure of the hypotenuse is 50.

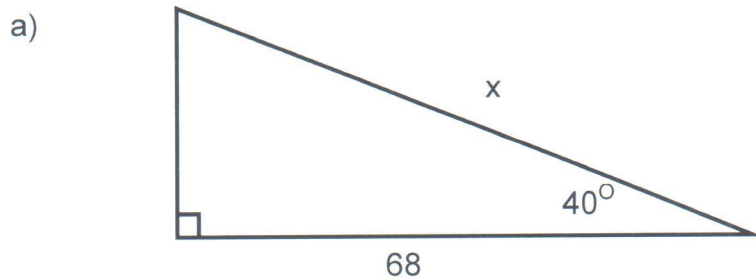
NOW DO PRACTICE EXERCISE 19

**Practice Exercise 19**

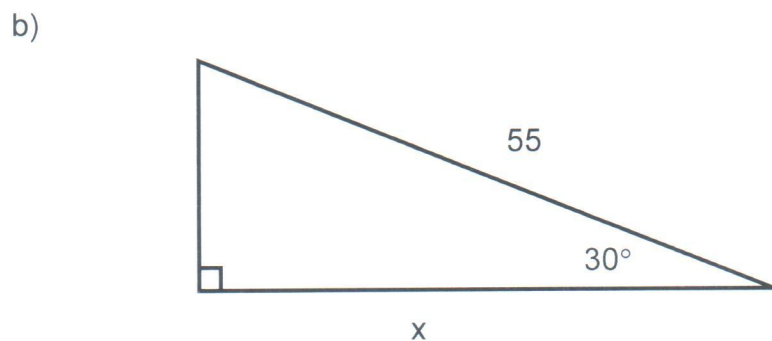
1. Using the trigonometric table for Cosine Ratio, Find the values of the following:

- a) $\cos 55^\circ$ _____
- b) $\cos 75^\circ$ _____
- c) $\cos 30^\circ$ _____
- d) $\cos 17^\circ$ _____
- e) $\cos 85^\circ$ _____
- f) $\cos 90^\circ$ _____

2. Solve for x in the following figures:



Answer: _____



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 20: The Tangent Ratio



You have learnt the primary trigonometric ratios in the previous lesson.



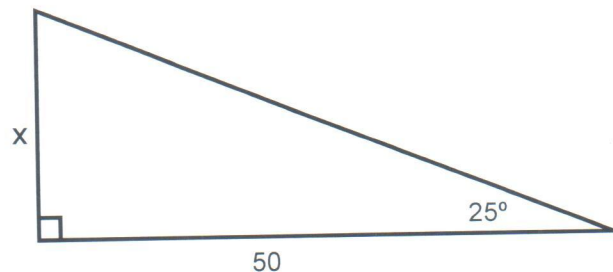
In this lesson, you will:

- define and explain the tangent ratio
- use trigonometric table to find the tangent ratio for a given angle
- find an angle and the length of a side of a right-angled triangle given its tangent ratio.

We have discussed in Lesson 17 the primary trigonometric ratios. In this lesson we will have a thorough study of the tangent ratio or the ratio formed out of the opposite side and adjacent side of a right triangle.

To use this ratio we must have at least given values of any two of the angle, adjacent and opposite sides of the right triangle.

Example 1



Solution:

Form the tangent ratio: $\tan 25^\circ = \frac{x}{50}$

Cross multiply: $(\tan 25^\circ)(50) = x$

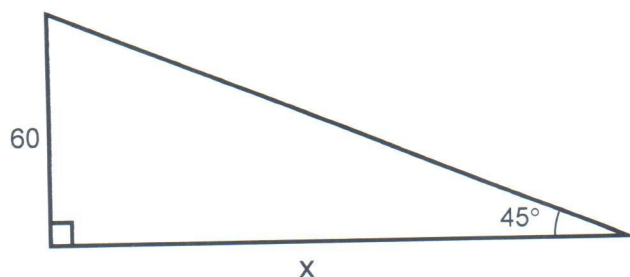
To get $\tan 25^\circ$, you can use a scientific calculator or you can use the trigonometric table for tangent function found on the next page.

Simplify $(0.466)(50) = x$
 $23.3 = x$

Therefore, the value of the opposite side is 23.3.

Trigonometry Table Tangent Ratio					
Radian	Degree	Tangent	Radian	Degree	Tangent
0.000	0	0.000	0.803	46	1.036
0.017	1	0.017	0.820	47	1.072
0.035	2	0.035	0.838	48	1.111
0.052	3	0.052	0.855	49	1.150
0.070	4	0.070	0.873	50	1.192
0.087	5	0.087	0.890	51	1.235
0.105	6	0.105	0.908	52	1.280
0.122	7	0.123	0.925	53	1.327
0.140	8	0.141	0.942	54	1.376
0.157	9	0.158	0.960	55	1.428
0.175	10	0.176	0.977	56	1.483
0.192	11	0.194	0.995	57	1.540
0.209	12	0.213	1.012	58	1.600
0.227	13	0.231	1.030	59	1.664
0.244	14	0.249	1.047	60	1.732
0.262	15	0.268	1.065	61	1.804
0.279	16	0.287	1.082	62	1.881
0.297	17	0.306	1.100	63	1.963
0.314	18	0.325	1.117	64	2.050
0.332	19	0.344	1.134	65	2.145
0.349	20	0.364	1.152	66	2.246
0.367	21	0.384	1.169	67	2.356
0.384	22	0.404	1.187	68	2.475
0.401	23	0.424	1.204	69	2.605
0.419	24	0.445	1.222	70	2.747
0.436	25	0.466	1.239	71	2.904
0.454	26	0.488	1.257	72	3.078
0.471	27	0.510	1.274	73	3.271
0.489	28	0.532	1.292	74	3.487
0.506	29	0.554	1.309	75	3.732
0.524	30	0.577	1.326	76	4.011
0.541	31	0.601	1.344	77	4.331
0.559	32	0.625	1.361	78	4.705
0.576	33	0.649	1.379	79	5.145
0.593	34	0.675	1.396	80	5.671
0.611	35	0.700	1.414	81	6.314
0.628	36	0.727	1.431	82	7.115
0.646	37	0.754	1.449	83	8.144
0.663	38	0.781	1.466	84	9.514
0.681	39	0.810	1.484	85	11.430
0.698	40	0.839	1.501	86	14.301
0.716	41	0.869	1.518	87	19.081
0.733	42	0.900	1.536	88	28.636
0.750	43	0.933	1.553	89	57.290
0.768	44	0.966	1.571	90	undefined
0.785	45	1.000			

Example 2



Solution:

Form the tangent ratio: $\tan 45^\circ = \frac{60}{x}$

Cross multiply: $x = \frac{60}{\tan 45^\circ}$

To get $\tan 45^\circ$, you can use a scientific calculator or you can use the trigonometric table for sine function found on the previous page.

Simplify $x = \frac{60}{1}$
 $x = 60$

Therefore, the value of the adjacent side is 60, it means that the right triangle is an isosceles right.

NOW DO PRACTICE EXERCISE 20

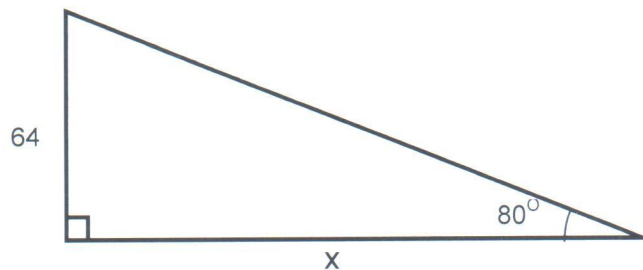
**Practice Exercise 20**

1. Using the trigonometric table for Tangent Ratio, Find the values of the following:

- a) $\tan 65^\circ$ _____
- b) $\tan 25^\circ$ _____
- c) $\tan 40^\circ$ _____
- d) $\tan 75^\circ$ _____
- e) $\tan 85^\circ$ _____
- f) $\tan 90^\circ$ _____

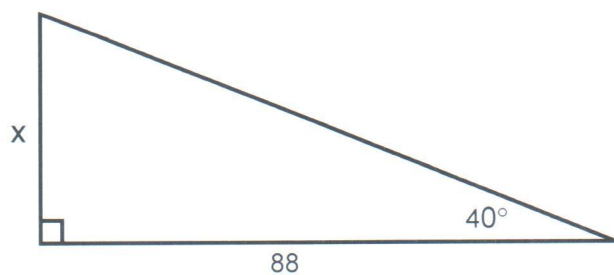
2. Solve for x in the following figures

a)



Answer: _____

b)



Answer: _____

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 21: Mixed Problem



You have learned about the primary trigonometric ratios.



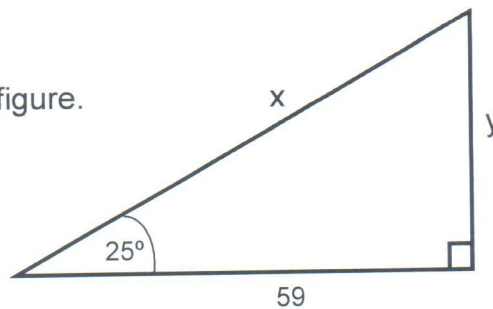
In this lesson, you will:

- use the three ratios to find lengths of sides and angles of right-angled triangle.

In this lesson, we will identify which of the three trigonometric ratios will be used to solve for the unknown measures of a right triangle.

Example 1

Solve for x and y in the figure.



Solution:

To solve for x we use the Cosine Ratio since x is the hypotenuse and one of the given values is the adjacent side.

Forming the ratio: $\cos 25^\circ = \frac{59}{x}$

Cross multiply: $x = \frac{59}{\cos 25}$

To get $\cos 25^\circ$, you can use a scientific calculator or you can use the trigonometric table for cos function found in Lesson 19.

Simplify $x = \frac{59}{0.906}$
 $x = 65.12$

To solve for y we use the Tangent Ratio since y is the opposite side and one of the given values is the adjacent side.

Form the tangent ratio: $\tan 25^\circ = \frac{y}{59}$

Cross multiply: $(\tan 25^\circ)(59) = y$

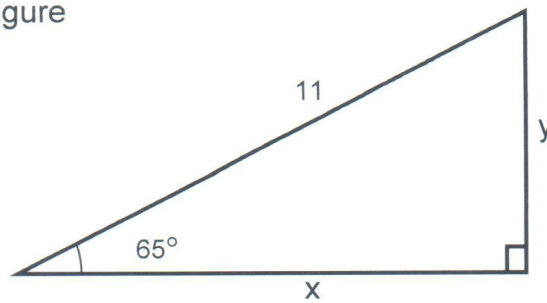
To get $\tan 25^\circ$, you can use a scientific calculator or you can use the trigonometric table for tangent function found in Lesson 20.

Simplify $(0.466)(59) = y$
 $27.494 = y$

Therefore, the value of the opposite side (y) is 27.49 and the value of the hypotenuse is 65.12.

Example 2

Solve for x and y in the figure



Solution:

To solve for x we use the Cosine Ratio since x is the adjacent side and one of the given values is the hypotenuse.

Form the cosine ratio: $\cos 65^\circ = \frac{x}{112}$

Cross multiply: $(\cos 65^\circ)(112) = x$

To get $\cos 65^\circ$, you can use a scientific calculator or you can use the trigonometric table for cosine function found in Lesson 19.

Simplify $(0.423)(112) = x$
 $47.38 = x$

To solve for y we use the Sine Ratio since y is the opposite side and one of the given values is the hypotenuse.

Form the sine ratio: $\sin 65^\circ = \frac{y}{112}$

Cross multiply: $(\sin 65^\circ)(112) = y$

To get $\sin 65^\circ$, you can use a scientific calculator or you can use the trigonometric table for cosine function found in Lesson 18.

Simplify $(0.906)(112) = y$
 $101.47 = y$

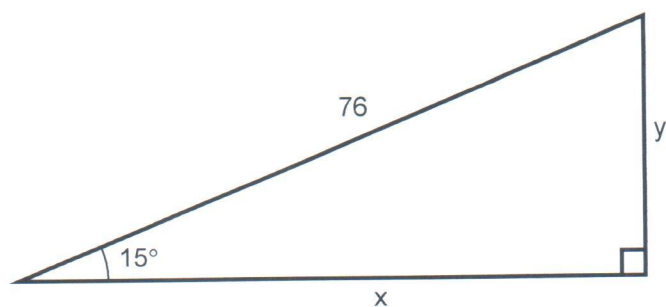
Therefore, the value of the opposite side (y) is 101.47 and the value of the adjacent side (x) is 47.38.

NOW DO PRACTICE EXERCISE 21

**Practice Exercise 21**

Solve for the values of x and y in the following figures:

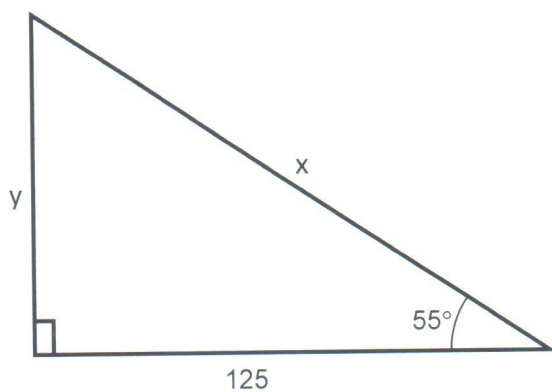
1.



Answer: $x =$ _____

$y =$ _____

2.



Answer: $x =$ _____

$y =$ _____

x

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

TOPIC 4: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- The word **Trigonometry** was derived from the Greek words **trigonon** meaning "triangle" (from tri- "three" + gonia "angle") and **metron** which means "a measure".
- The **initial side** is the leg of an angle which always lies horizontally. The initial side does not move. It always sticks to its position.
- The **terminal side** is the leg of an angle which determines the rotation of the angle. This side can move in either clockwise or anti-clockwise direction.
- For positive angles, the rotation is anti-clockwise or counter-clockwise.
- Negative angles have opposite rotation which is clockwise.
- **The Primary Trigonometric Ratios are:**

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$$

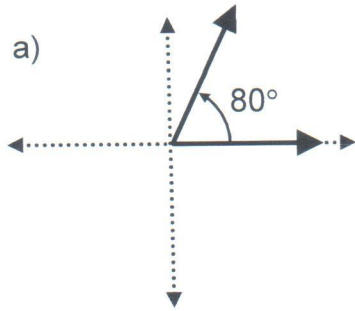
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

REVISE LESSONS 17 TO 21. THEN DO TOPIC TEST 3 IN ASSIGNMENT BOOK 5.

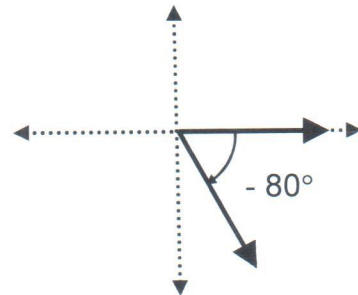
ANSWERS TO PRACTICE EXERCISES 17-21

Practice Exercise 17

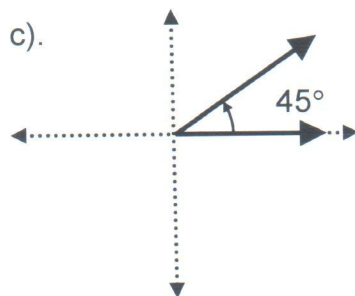
1. a)



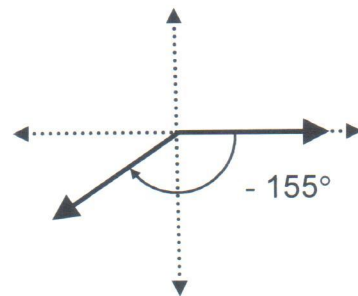
b)



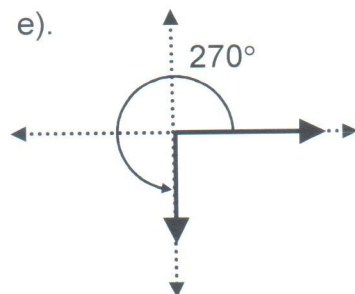
c).



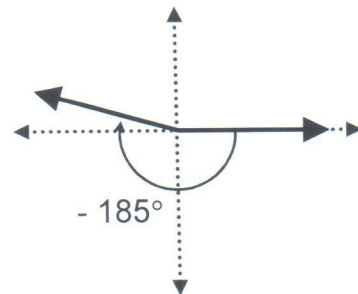
d)



e).



f)



2. a) $\sin \theta = \frac{-4}{5}$
 $\cos \theta = \frac{-3}{5}$
 $\tan \theta = \frac{4}{3}$

b) $\sin \theta = \frac{-5}{\sqrt{29}}$
 $\cos \theta = \frac{2}{\sqrt{29}}$
 $\tan \theta = \frac{-5}{2}$

3. a) $\sin A = \frac{-4}{5}$ $\cos A = \frac{-3}{5}$ $\tan A = \frac{-4}{-3} = \frac{4}{3}$

b) $\sin A = \frac{-5}{\sqrt{29}}$ $\cos A = \frac{2}{\sqrt{29}}$ $\tan A = \frac{-5}{2}$

Practice Exercise 18

1. a) 0.707
b) 0.259
c) 0.50
d) 0.940
e) 0.996
f) 1
2. a) 23.095 b) 103.92
-

Practice Exercise 19

1. a) 0.573
b) 0.259
c) 0.866
d) 0.956
e) 0.087
f) 0
2. a) 88.77 b) 47.63
-

Practice Exercise 20

1. a) 2.145
b) 0.466
c) 0.839
d) 3.732
e) 11.43
f) undefined
2. a) 11.285 b) 73.832
-

Practice Exercise 21

1. a) $x = 73.416$ $y = 19.684$
b) $x = 217.770$ $y = 178.50$
-

END OF UNIT 5

NOW YOU MUST COMPLETE ASSIGNMENT 5

REFERENCES

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<http://media-cache-ec0.pinimg.com/236x/b7/3b/d3/b73bd31dfd804474aed85167a210efc0.jpg>

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50 toea coin

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http://mathforum.org/mathimages/imgUpload/More_difficult_problems.jpg

100 Kina Note

http://api.ning.com/files/5V0Mkg4QBt5YKR8Pa8BNiGdu57fv5C8Bk6fapONbcqP0t7aWkeOJDghng3AzOYNfofgcyYHbAM8a4FW*XkflSNfVhIFAYKro/PNGHybird100kinaB.PNG000123.jpg

Soccer Ball

<http://www.regentsprep.org/regents/math/geometry/gg2/soccerball.jpg>

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FODE PROVINCIAL CENTRES

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21	HELA	P. O. Box 63, Tari	72228141	77522861	Mr Ogai John	72229083
22	JIWAKA	c/- FODE Hagen	72228143	77522862	Joseph Walep	72229085

SUBJECT AND GRADE TO STUDY

Grade Levels	Subjects
Grades 7 and 8	1. English
	2. Mathematics
	3. Science
	4. Social Science
	5. Making a Living
	6. Personal Development
	7. English
Grades 9 and 10	1. English
	2. Formal Mathematics
	3. Practical Mathematics
	4. Science
	5. Social Science
	6. Commerce
	7. Design and Technology- Computing
	8. Personal Development
Grades 11 and 12	1. English
	• Applied English
	• Language and Literature
	2. Mathematics
	• Mathematics A
	• Mathematics B
	3. Science
	4. Social Science
	• History
	• Geography
	• Economics
	5. Business Studies
	6. Personal Development
	7. ICT

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) courses.
- For Grades 9 and 10, you must study English, Mathematics, Science, Personal Development, Social Science and Commerce. Design and Technology-Computing is optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) courses to be certified. Your Provincial Coordinator or Supervisor will give you more information regarding each subject.

Certificate in Matriculation

CORE COURSES

Basic English
English 1
English 2
Basic Maths
Maths 1

Maths 2

History of Science & Technology

OPTIONAL COURSES

Science Streams: Biology

Chemistry, Physics and Social Science Streams:

Geography, Introduction to Economics and Asia and the Modern World

REMEMBER:

You must successfully complete 8 courses; 5 compulsory and 3 optional